2

Since $X \sim U(1,3) \Rightarrow EX = \int_1^3 x f(x)\,dx =$

$$= \int_1^3 x \cdot \frac{1}{2}\,dx = \frac{1}{2}\int_1^3 x\,dx = \frac{1}{2}\,\frac{x^2}{2}\Big/_1^3 = \frac{1}{2}\cdot\left(\frac{9}{2} - \frac{1}{2}\right)$$

$$= \frac{1}{2}\cdot 4 = 2 \quad \text{OR Directly} \quad EX = \frac{b+a}{2} = \frac{3+1}{2} = 2$$

$$\therefore E(A) = \frac{1}{2}\cdot EX \cdot EY = \frac{1}{2}\cdot 2 \cdot 2 = 2.$$

$$\operatorname{Var} A = E(A^2) - (EA)^2$$

$$EA^2 = E\left(\frac{XY}{2}\right)^2 = E\left(\frac{X^2 \cdot Y^2}{4}\right) = \frac{1}{4}\,EX^2 \cdot Y^2 \;\underline{\underline{=}}$$

X, Y indep $\Rightarrow$ X^2, Y^2 — " —

$$\frac{1}{4}\,EX^2 \cdot EY^2.$$

But $\operatorname{Var} X = EX^2 - (EX)^2 \Rightarrow EX^2 = \operatorname{Var} X + (EX)^2.$

Again $X \sim U(1,3) \Rightarrow \operatorname{Var} X = \frac{(b-a)^2}{12} = \frac{(3-1)^2}{12} = \frac{4}{12} = \frac{1}{3}$

and $EX^2 = \operatorname{Var} X + (EX)^2 = \frac{1}{3} + (2)^2 = \frac{1}{3} + 4 = \frac{13}{3}.$

and hence

$$EA^2 = \frac{1}{4}\cdot\frac{13}{3}\cdot\frac{13}{3} = \frac{169}{36}$$

$$\therefore \rho_{X_1,X_2} = \frac{0.25+0.5t-0.25}{\sqrt{0.25 * 0.25}} = \frac{0.5t}{0.25} = 2t$$

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4) $$f(x) = \begin{cases} (\theta+1)x^{\theta}, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta) = \prod_{i=1}^{n} (\theta+1)x_i^{\theta} = (\theta+1)^n \cdot \prod_{i=1}^{n} x_i^{\theta}$$

$$\ln L(\theta) = \ln\left[(\theta+1)^n \cdot \prod_{i=1}^{n} x_i^{\theta}\right] = n\ln(\theta+1) + \sum_{i=1}^{n} \theta \ln x_i$$

$$\frac{\partial \ln L(\theta)}{\partial \theta} = \frac{n}{\theta+1} + \sum_{i=1}^{n} \ln x_i = 0 \Rightarrow$$

$$\frac{n}{\theta+1} = -\sum_{i=1}^{n} \ln x_i \Rightarrow \frac{\theta+1}{n} = -\frac{1}{\sum_{i=1}^{n} \ln x_i} \Rightarrow$$

$$\theta+1 = -\frac{n}{\sum_{i=1}^{n} \ln x_i} \Rightarrow \hat{\theta} = -1 - \frac{n}{\sum_{i=1}^{n} \ln x_i}$$

6

	Observed Freq.
$P(SF) = 0.1$	$O_1 = 13$
$P(HF) = 0.05$	$O_2 = 10$
$P(OE) = 0.25$	$O_3 = 42$
$P(SO) = 04$	$O_4 = 65$
$P(Rest) = 0.2$	$O_5 = 20 = 150 - 130$

Expected freq $= 150 * P$

SF $\rightarrow$ $15 = E_1$

HF $\rightarrow$ $150 * 0.05 = 7.5 = E_2$

OE $\rightarrow$ $150 * 0.25 = 37.5 = E_3$

SO $\rightarrow$ $150 * 0.4 = 60 = E_4$

Rest $\rightarrow$ $150 * 0.2 = 30 = E_5$

$$\chi^2 = \sum_{i=1}^{5} \frac{(O_i - E_i)^2}{E_i} = \frac{(13-15)^2}{15} + \frac{(10-7.5)^2}{7.5} + \frac{(42-37.5)^2}{37.5} +$$

$$+ \frac{(65-60)^2}{60} + \frac{(20-30)^2}{30} = 5.39$$

Choose $\alpha = 0.05$ $\quad$ d.f $= k - p - 1 = 5 - 0 - 1 = 4$.

$\chi^2_{0.05, 4} = 9.49$

Since $\chi_0^2 = 5.39 < \chi^2_{0.05, 4} = 9.49$ we are unable to reject the null hypothesis H_0: that the stated percentages are correct.

(8) X = # of times we throw a diee until we get 2 successes. (8)

where success is to get a face of 6.

Then X is a negative binomial (tosses where independent Bernoulli trials) with $p=\frac{1}{6}$ and $r=2$.

$$f(x)=\binom{x-1}{r-1}(1-p)^{x-r}\cdot p^{r}=\binom{x-1}{1}\left(\frac{5}{6}\right)^{x-2}\cdot\frac{1}{6}^{2},\ x=2,3,\dots$$

(9) a) $\hat{\beta}_0=\bar{y}-\hat{\beta}_1\bar{x}$

$$\hat{\beta}_1=\frac{\sum x_iy_i-\frac{\sum y_i\cdot\sum x_i}{n}}{\sum x_i^2-\frac{(\sum x_i)^2}{n}}=\frac{285.625-\frac{170*16.75}{10}}{28.64-\frac{(16.75)^2}{10}}=$$

$$=\frac{0.8750}{0.5838}=1.4988\approx\underline{\underline{1.5}}$$

$$\hat{\beta}_0=\frac{170}{10}-1.5\cdot\frac{16.75}{10}=17-2.5125=\underline{\underline{14.4875}}$$

$\therefore\ \hat{Y}=14.4875+1.5x.$

$t_{0.005,8} = 3.355$

Since $|t_0| = 8.339 > t_{\alpha/2, n-2}$ we reject H_0.
That means the regression is important, and ~~exp~~ there is a linear relation between Y and x.

d) If $\varepsilon_i \sim N(0, \sigma^2)$, since $\hat{Y}_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

Then $Y_i \sim N(\beta_0 + \beta_1 x_i, \sigma^2)$

$Y_i \sim$ normally distributed, because $\varepsilon_i \sim N(0, \sigma^2)$.

$EY_i = E(\beta_0 + \beta_1 x_i + \varepsilon_i) = E(\beta_0 + \beta_1 x_i) + E(\varepsilon_i) =$

$= \beta_0 + \beta_1 x_i$

$Var(Y_i) = Var(\beta_0 + \beta_1 x_i + \varepsilon_i) = Var(\beta_0 + \beta_1 x_1) + Var(\varepsilon_i) =$

$= 0 + \sigma^2 = \underline{\underline{\sigma^2}}$