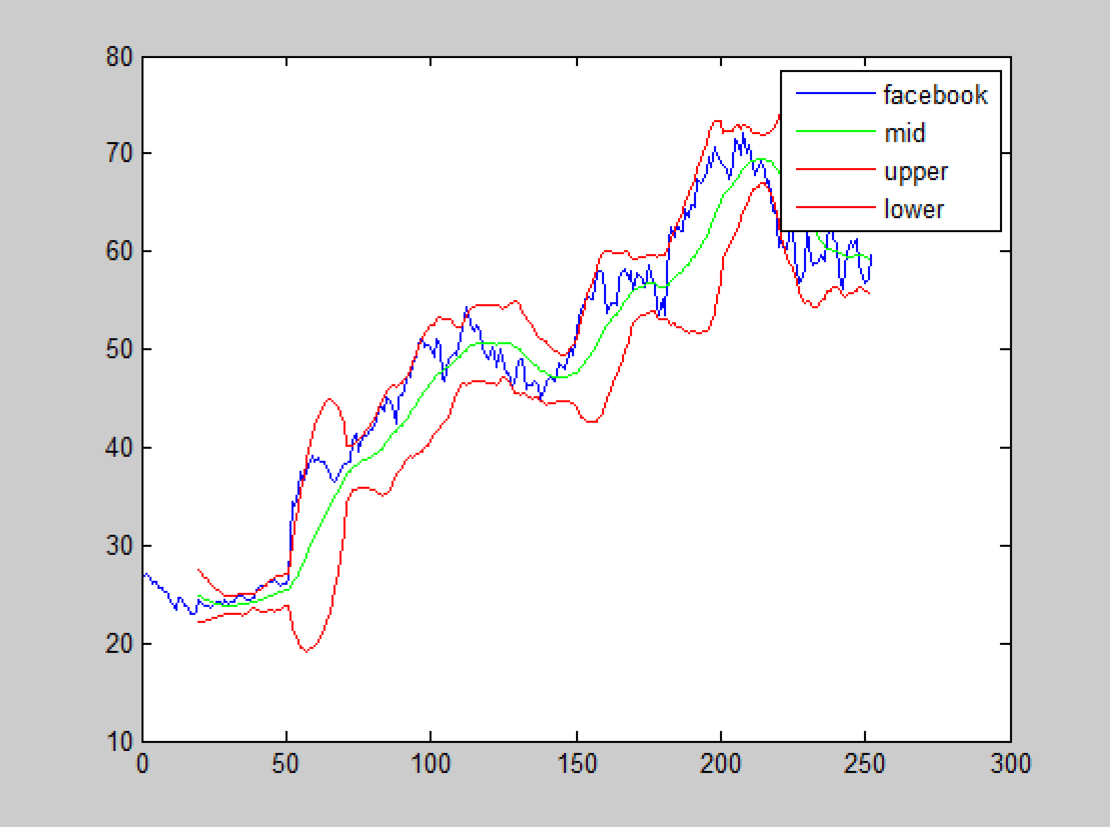
Financial Times Series

Lecture 13

Bollinger bands

- For the adjusted closing prices of the Facebook stock the Bollinger bands are given below



Bollinger bands

- Bollinger bands is a technical analysis tool to be used as an indicator for when to buy and sell
- The mid band is a simple moving average of the last, typically twenty, observations
- The lower and upper bands are the mid band plus minus some, typically two, standard deviations of the last, typically twenty, observations

Strategies

- One strategy is to buy when the price touches the lower band and to sell when the price touches the mid band
- Another is to buy when the price touches the upper band and to sell when it touches the mid or the lower band
- A ”squeeze” is supposed to indicate that something is about to happen. However, a squeeze gives no clue on the direction of the movement to come…

JC Penney


Bollinger Bands for JC Penney May 19th 2012-May 18th 2014
0
5
10
15
20
25
30
35
0
50
100
150
200
250
300
350
400
450
500

Markov Switching Multifractal

- In a general Markov switching model univariate returns are given by

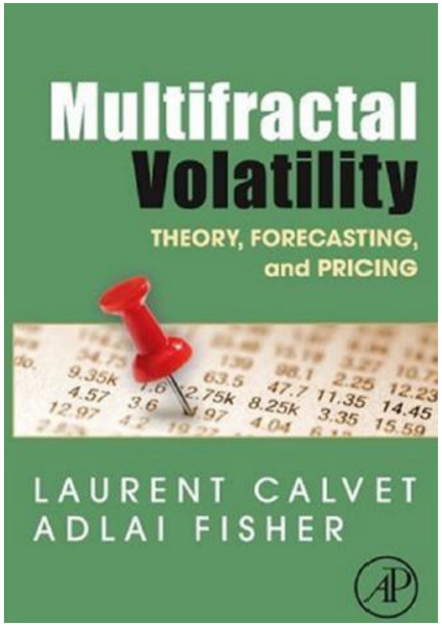
$$r_t = \mu(M_t) + \sigma(M_t)\varepsilon_t$$

where $\mu(M_t)$ and $\sigma(M_t)$ are drifts and volatilities as functions of the latent (unobservable that is) state variable M_t and ε_t is standard normal white noise

Markov Switching Multifractal

- Previously we have seen the ”bull bear model” where M_t is either bull or bear so that in the bull state the drift is positive and the volatility is low and in the bear state the drift is negative and the volatility is high

- The MSM (Calvet and Fisher) typically has more than two states and the aim is to capture all (or most) empirical facts for return data such as volatility persistence, skewness and heavy tails



The basic idea

- The latent state is the product of the components of a vector representing factors that drive the economy

$$M_t = \prod_{k=1}^{\bar{k}} M_{k,t}$$

- At each time point t the factor $M_{k,t}$ is drawn from a distribution/r.v. $M \geq 0, E(M) = 1$ with probability γ_k and stays at its previous $(t-1)$ value with probability $1 - \gamma_k$

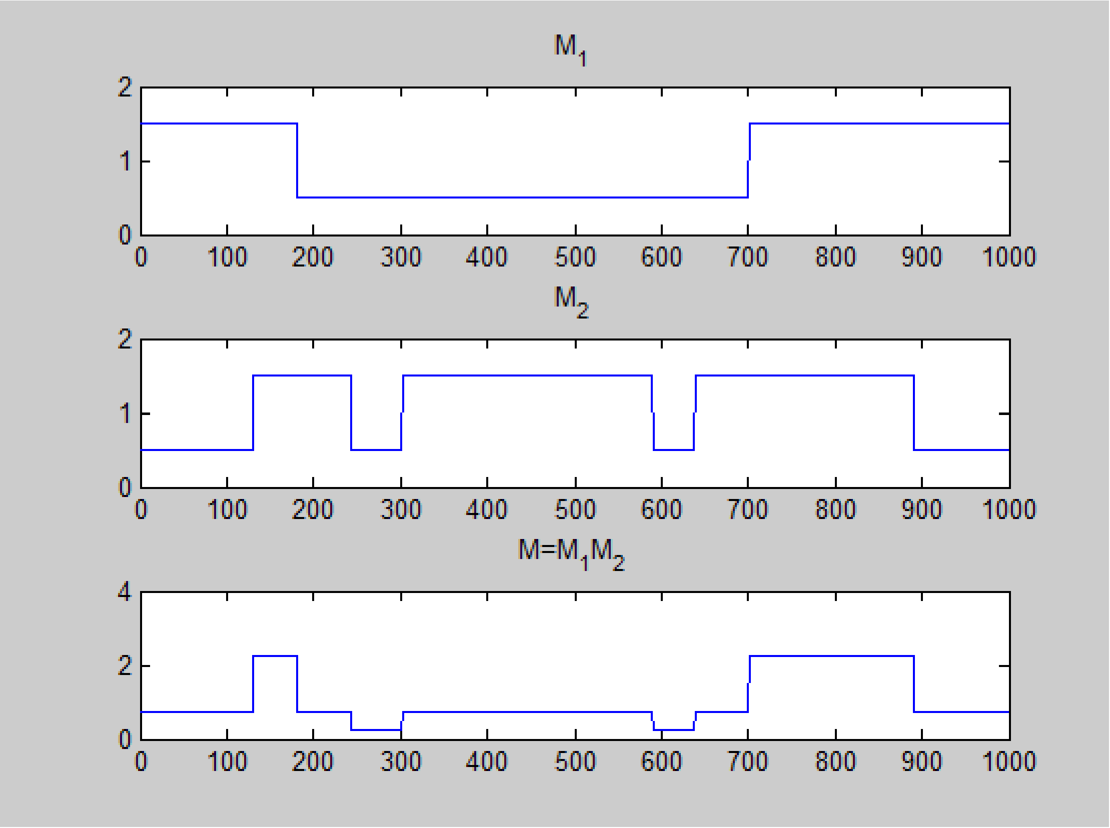
The basic idea

- The factors $M_{1,t}, \dots, M_{\bar{k},t}$ are mutually independent and switch independently of each other where $M_{1,t}$ has the lowest switching frequency and $M_{\bar{k},t}$ has the highest switching frequency where we assume that

$$\gamma_k = 1 - (1 - \gamma_1)^{b^{k-1}}$$

where $0 < \gamma_1 < 1$ and $b > 1$

Simple example with two two-state factors



Binomial MSM

- In the above example the random variable M takes the values m_0 or $m_1 = 2 - m_0$ w.p. 0.5 (remembering the restriction $E(M) = 1$)

- As we can imagine the restrictions and relations for the switching frequencies and number of states for each factor helps keeping the number of parameters low

Volatility model

- The volatility in the MSM is given by

$$\sigma(M_t) = \bar{\sigma}\left(\prod_{k=1}^{\bar{k}} M_{k,t}\right)^{1/2}$$

where $\bar{\sigma}$ is the unconditional standard deviation of the returns

- Under the binomial framework the full set of parameters are

$$(m_0, \bar{\sigma}, b, \gamma_{\bar{k}})$$

Estimation

- Under the binomial framework the number of possible states are $d = 2^{\bar{k}}$ which we define as $\left(m^1, \dots, m^d\right)$ corresponding to a transition probability matrix $A = \left[a_{i,j}\right]_{1 \le i,j \le d}$ where the elements are given by

$$a_{i,j} = P\left(M_{t+1} = m^j | M_t = m^i\right) = \prod_{k=1}^{\bar{k}} \left((1 - \gamma_k) 1_{\{m^i{}_k = m^j{}_k\}} + \frac{\gamma_k}{2}\right)$$

- Conditional on the volatility state the density of the mean corrected return r_t is

$$\varphi\left(\frac{r}{\sigma(m^i)}\right)$$

where φ denotes the density of a standard normal r.v.

Estimation

- The states are not observable but we may use the conditional probabilities

$$\Pi^j{}_t = P\left(M_t = m^j | r_1, \dots, r_t\right)$$

and the vector $\Pi_t = \left(\Pi^1{}_t, \dots, \Pi^d{}_t\right)$ may be computed recursively using Bayes' rule

$$\Pi_t = \frac{\omega(r_t) \odot (\Pi_{t-1} A)}{[\omega(r_t) \odot (\Pi_{t-1} A)]\mathbf{1}'}$$

where $\omega(r_t) = \left(\varphi\left(\frac{r_t}{\sigma(m^1)}\right), \dots, \varphi\left(\frac{r_t}{\sigma(m^d)}\right)\right)$ (for the standard normal density φ) and $\odot$ is the Hadamard product

Estimation

- To start the recursion one may use the ergodic (steady-state) distribution of the Markov chain as Π_0

- In the binomial case and by independence of the factors $M_{k,t}$ this distribution is given by (for all j)

$$2^{-\bar{k}} \prod_{l=1}^{\bar{k}} \left(m_0 1_{\{m^j{}_l = m_0\}} + (2 - m_0) 1_{\{m^j{}_l = 2 - m_0\}} \right)$$

Estimation

- Closed form log-likelihood for the binomial model;

$$l(m_0, \bar{\sigma}, b, \gamma_{\bar{k}} | r_1, \dots, r_T) = \sum_{t=1}^{T} \ln \left[\omega(r_t) \cdot (\Pi_{t-1} A)\right]$$

where $\cdot$ denotes scalar product

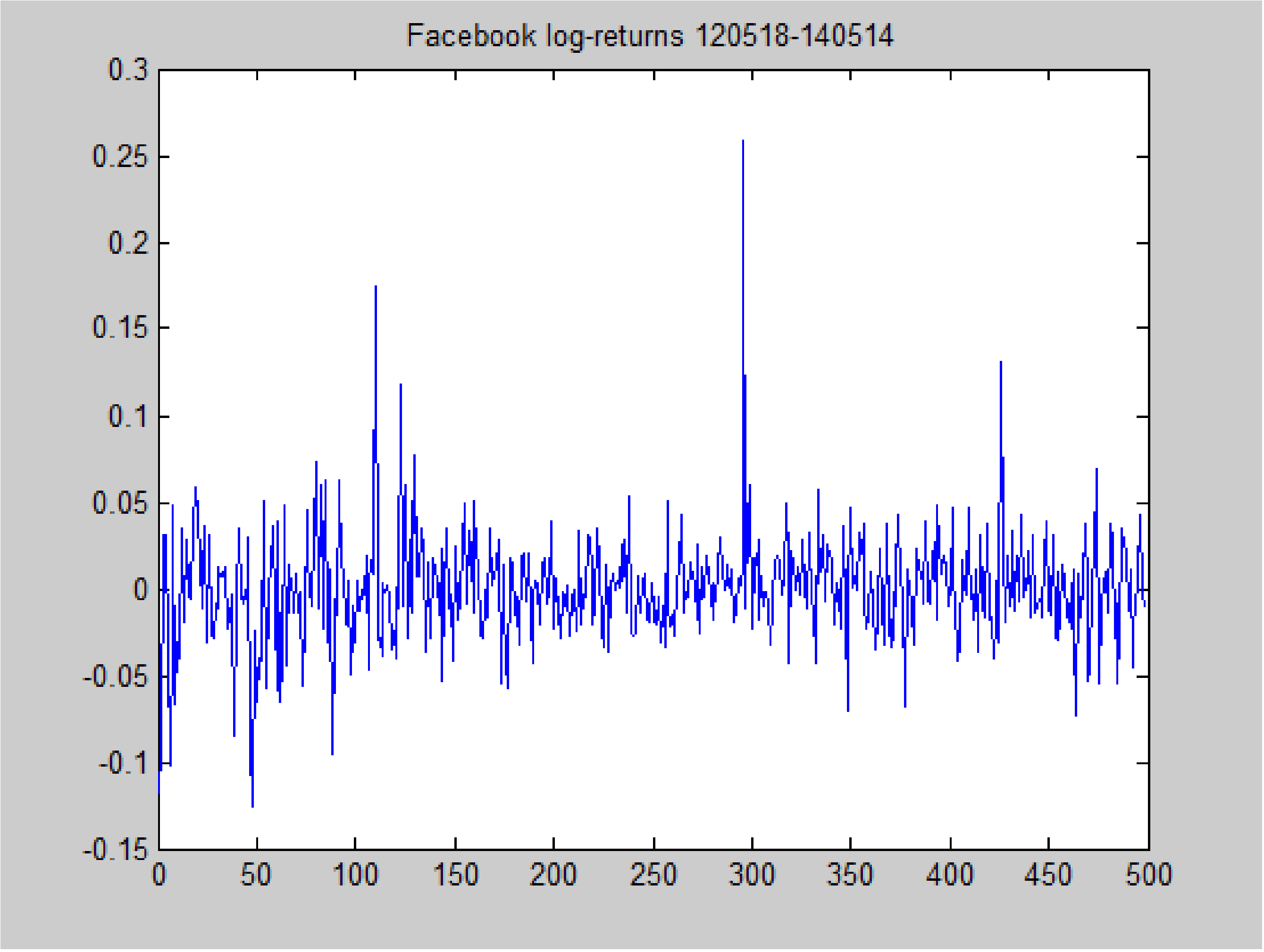
- Matlab codes for estimation of the binomial model are available at https://studies2.hec.fr/jahia/Jahia/calvet

Estimation

- How to choose the number of factors $\bar{k}$?

- Noting that the number of parameters is constant (and equal to 4) checking information criteria like AIC and BIC is equivalent to checking the value of the log-likelihood at the maximum

For Facebook data



For Facebook data

$\bar{k}$	1	2	3	4	5	6
$\hat{b}$	1.500	50.00	16.66	20.89	7.011	50.00
$\hat{m}_0$	1.723	1.606	1.493	1.489	1.423	1.605
$\hat{\gamma}_{\bar{k}}$	0.069	0.958	1.000	1.000	1.000	0.952
$\hat{\bar{\sigma}}$	0.700	0.610	0.558	0.461	0.683	3.898
l	1049	1054	1054	1053	1053	1051

So it seems that two or three factors should suffice to describe the volatility of the Facebook stock

Forecasting

- We have, with $F_{t-1} = \{r_1, \dots, r_{t-1}\}$

$$E[\sigma^2{}_t | F_{t-1}] = \bar{\sigma}^2 \sum_{i,j=1}^{d} m^j \Pi^i{}_{t-1} a_{ij}$$

since

$$E[M_t | F_{t-1}] = \sum_{j=1}^{d} m^j P\left(M_t = m^j | F_{t-1}\right)$$

where

$$P\left(M_t = m^j | F_{t-1}\right) = \sum_{i=1}^{d} P\left(M_t = m^j, M_{t-1} = m^i | F_{t-1}\right)$$

and

$$P\left(M_t = m^j, M_{t-1} = m^i | F_{t-1}\right) = P\left(M_{t-1} = m^i | F_{t-1}\right) P\left(M_t = m^j | M_{t-1} = m^i\right) = \Pi^i{}_{t-1} a_{ij}$$

Forecasting

	Mincer–Zarnowitz		Restricted $\gamma_0 = 0, \gamma_1 = 1$	
	γ_0	γ_1	MSE	R^2
Deutsche Mark/U.S. Dollar				
Binomial MSM	0.098	0.703	0.7263	0.041
	(0.072)	(0.126)		
GARCH	0.153	0.622	0.7304	0.035
	(0.061)	(0.105)		
MS-GARCH	0.042	0.740	0.7296	0.037
	(0.080)	(0.130)		
Japanese Yen/U.S. Dollar				
Binomial MSM	0.028	0.772	1.6053	0.053
	(0.090)	(0.117)		
GARCH	0.172	0.668	1.6137	0.048
	(0.075)	(0.105)		
MS-GARCH	0.080	0.709	1.6141	0.048
	(0.084)	(0.109)		

Forecasting

British Pound/U.S. Dollar

Binomial MSM	0.053 (0.049)	0.715 (0.100)	0.5081	0.057
GARCH	0.085 (0.044)	0.751 (0.098)	0.4980	0.076
MS-GARCH	0.017 (0.051)	0.814 (0.108)	0.4997	0.072

Canadian Dollar/U.S. Dollar

Binomial MSM	0.015 (0.016)	0.905 (0.156)	0.0345	0.051
GARCH	0.033 (0.012)	0.679 (0.111)	0.0348	0.042
MS-GARCH	0.025 (0.013)	0.785 (0.124)	0.0344	0.055