CHALMERS | UNIVERSITY OF GOTHENBURG

Financial Time Series TMS087/MSA410 – LP4 2014/15

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“Theoretical” Questions

1. Compute the ACF and the PACF of the $\mathrm{AR}(2)$ process

$$X_t = 0.8X_{t-2} + Z_t,$$

where $(Z_t, t \in \mathbb{Z}) \sim \mathrm{WN}(0, \sigma^2)$.

2. Determine which of the following ARMA processes are causal and which of them are invertible, where $(Z_t, t \in \mathbb{Z}) \sim \mathrm{WN}(0, \sigma^2)$:

a) $X_t - 0.2X_{t-1} - 0.48X_{t-2} = Z_t$

b) $X_t + 0.6X_{t-1} = Z_t + 1.2Z_{t-1}$

c) $X_t + 1.6X_{t-1} = Z_t - 0.4Z_{t-1} + 0.04Z_{t-2}$

3. The Gaussian likelihood for an ARMA process X is given by

$$L(\phi, \theta, \sigma^2) = (2\pi\sigma^2)^{-n/2} \prod_{j=1}^{n} r_{j-1}^{-1/2} \exp\left(-(2\sigma^2)^{-1} \sum_{j=1}^{n} r_{j-1}^{-1} (X_j - \hat{X}_j)^2 \right), \tag{1}$$

where $\hat{X} = (\hat{X}_j, j = 1, \ldots, n)$ denotes the series of one-step predictors. The goal of this problem is to derive maximum likelihood estimators for σ^2, ϕ, and θ, which will be divided into two steps.

a) Differentiate $\ln L(\phi, \theta, \sigma^2)$ with respect to σ^2 and use this to maximize (1) with respect to σ^2 to obtain the maximum likelihood estimator

$$\hat{\sigma}^2 = n^{-1} S(\hat{\phi}, \hat{\theta}),$$

where

$$S(\hat{\phi}, \hat{\theta}) = \sum_{j=1}^{n} r_{j-1}^{-1} (X_j - \hat{X}_j)^2$$

and $\hat{\phi}$ and $\hat{\theta}$ are the estimators of ϕ and θ which are derived in b).

Please turn!

b) Use your result from a) and show that the maximization of (1) is equivalent to the minimization of

$$\ell(\phi, \theta) = \ln(n^{-1} S(\phi, \theta)) + n^{-1} \sum_{j=1}^{n} \ln r_{j-1}$$

where we denote the derived estimators by $\hat{\phi}$ and $\hat{\theta}$ in a).

Purpose: The purpose of these questions is to get an idea of possible exam questions and especially how these should be written down.

"Deadline": The solutions will be discussed in class Friday, May 22, 2015

Webpage: `http://www.math.chalmers.se/Stat/Grundutb/CTH/tms087/1415/`

Requirement: Solving these questions is completely voluntary but highly recommended in order to be prepared for the discussion in class and for the exam.