Problem 1

Let a time series model $X := (X_t, t \in \mathbb{Z})$ be given by

$$X_t = \phi X_{t-1} + Z_t$$

for $t \in \mathbb{Z}$, where $(Z_t, t \in \mathbb{Z})$ is $\mathrm{WN}(0, \sigma^2)$ and $|\phi| \neq 1$. Assume that the stochastic process satisfying this model is stationary.

(a) Define the mean, the variance, and the autocovariance function of X and compute them.

(b) Define a causal time series and derive for which ϕ X is causal.

(c) Define an invertible time series and derive for which ϕ X is invertible.

(d) Consider ϕ such that the time series is causal and let two observation x_1 and x_2 be given with $|x_1| \neq |x_2|$. Derive the maximum likelihood estimates $\hat{\phi}$ and $\hat{\sigma}^2$ of ϕ and σ^2.

(e) Derive the best linear predictor of X_3 for the given observations x_1 and x_2 according to the fitted model from the preceding question.

(f) Compute the mean squared error of the best linear predictor of X_3. Are all errors that you made in the prediction process included?

(44 points)

Problem 2

Assume that you are asked by a customer for advice how to act in the future. The customer gives you n observations that should be turned into predictions of the future behavior and the risk involved in the forecasts. The goal of this problem is to describe the process from observations to forecasts as it was discussed in the lecture. It is not necessary to write a text but a graphical presentation and bullet points are sufficient. It is more important that the presentation is structured such that the instructions can be easily followed. You are free in your presentation but if you have problems finding a structure you might want to consider the categories "testing", "modeling", and "forecasting". Do not forget to mention the used methods with the main ideas behind them to solve the task.

(16 points)