2017-01-03

MSA410 & TMS087
Financial Time Series

Please make sure before you start:

- The exam is on *January 3, 2017, 8:30 – 12:30*.
- The examiner is *Annika Lang*, Applied Mathematics & Statistics, Chalmers, phone: 0317725356.
- I visit the exam at 9:30 and 11:00.
- You are *allowed* to use during the exam four pages (two sheets double-sided) of handwritten notes and a simple calculator.
- The maximum number of points that can be achieved is 60. You need *30 points* to pass the exam (for GU: 30 points for G and 45 points for VG, for Chalmers 30 points for 3, 40 points for 4, and 50 points for 5).
- Read all problems carefully before you start to work on the exam.
- Write your solutions in detail and readable. If you use theorems, lemmas, definitions, etc. from the lecture, cite the precise results. Missing details in your arguments lead to point deductions.

Problem 1

Let a time series model $X := (X_t, t \in \mathbb{Z})$ be given by

$$X_t := Y_t(Z_t + Z_{t-1})$$

for $t \in \mathbb{Z}$, where $Y := (Y_t, t \in \mathbb{Z})$ and $Z := (Z_t, t \in \mathbb{Z})$ are independent, stationary time series. Furthermore, Z is $\mathrm{WN}(0, \sigma_Z^2)$ and Y has autocovariance function γ_Y given by $\gamma_Y(h) := 2^{-|h|}$ for $h \in \mathbb{Z}$.

(a) Define a stationary time series and white noise. Show that X is stationary. *(8.5 points)*

(b) Compute the autocovariance function of X. *(1 point)*

(c) Define a moving average process of order q and compute the mean and the autocovariance function for an MA(1) process. *(3.5 points)*

(d) Assume from here on that $\mathbb{E}(Y_t) = 0$. Show that X is an MA(1) process and derive the two parameters. (*Hint:* Continue with the dummy parameters θ and σ^2 if you do not solve this problem.) *(6.5 points)*

(e) Define a causal time series and show if X is causal. *(2 points)*

(f) Define an invertible time series and show if X is invertible. *(4 points)*

(g) Write down the innovations algorithm for an MA(1) process. *(4 points)*

(h) Let two observations x_1 and x_2 be given with $x_1 \neq x_2$. Compute the sample mean and the sample autocovariance function. *(3.5 points)*

(i) Estimate the model parameters θ and σ^2 of the MA(1) process by $\hat{\theta}$ and $\hat{\sigma}^2$ from the innovations algorithm. *(4 points)*

(j) Use the innovations algorithm to compute the best linear predictor of X_3 for the given observations x_1 and x_2 according to the fitted model. *(11.5 points)*

(k) Compute the mean squared error of the best linear predictor of X_3. Are all errors that you made in the prediction process included? *(2.5 points)*

(l) Use the definition of the best linear predictor to compute the best linear predictor of X_3 directly. What do you observe with respect to your previous results? Explain your observation. (*Hint:* The relation $\mathbb{E}(X_i(X_{n+1} - b_{n+1}^l(X^n))) = 0$ for $i = 1, \dots, n$ might be helpful.) *(9 points)*