CHALMERS | UNIVERSITY OF GOTHENBURG

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Financial Time Series TMS087/MSA410 – LP4 2016/17

Project 1

The file *gaussian.mat* contains samples of the stationary Gaussian process

$$X = (X_t, t \in \mathbb{Z}) \tag{1}$$

with mean μ and autocovariance function (ACVF) γ. The vectors x and y within the file contain two independent samples $(x_i, i = 1, \dots, 1000)$ of the process (1) and each column of the 10×100-matrix Z also contains an independent sample $(x_i, i = 1, \dots, 10)$ of (1). You are to work with this data set using MATLAB. To import the contents of the mat-file, make sure your current folder in MATLAB contains the file and write

```
data = importdata('gaussian.mat');
```

The contents can then be accessed by writing `data.x` and so on.

1. (4 points) Denote by $\bar{X}_n$ the sample mean of X for a sample of size n. Show that

$$\bar{X}_n - \mu \sim \mathcal{N}\left(0, n^{-1} \sum_{|h|<n} (1 - n^{-1}|h|)\gamma(h)\right).$$

Hint: The distribution of a Gaussian random variable is completely determined by its mean and variance.

2. (2 points) Use the first sample, i.e. the vector x, to estimate and plot the sample ACVF $\hat{\gamma}(h)$ for X and $h = 0, \dots, 15$. Also estimate the mean μ.

3. (3 points) Assume that

$$\bar{X}_n - \mu \sim \mathcal{N}\left(0, n^{-1} \sum_{|h|<n} (1 - n^{-1}|h|)\hat{\gamma}(h)\right).$$

Use the 100 independent samples in Z to form 100 observations of $\bar{X}_{10}$ denoted by $(\bar{X}_{10}^i, i = 1, \dots, 100)$. Derive theoretically a 95% confidence interval for a Gaussian random variable with mean zero and variance $n^{-1} \sum_{|h|<n}(1 - n^{-1}|h|)\hat{\gamma}(h)$, where

Please turn!

$n = 10$ and $\hat{\gamma}$ as computed in the previous exercise. (*Hint: For* $Z \sim \mathcal{N}(0,1)$ *it holds that* $P(|Z| < 1.96) = 0.95$.*)* For each $\bar{X}_{10}^i$, $i = 1, \dots, 100$, use this result to compute a 95% confidence interval for μ. Check for each of the 100 intervals if the μ you estimated in the previous exercise falls within it or not. How many of them do? Is this in accordance with what you expect?

4. (4 points) Compute all possible one-step ahead linear forecasts

$$(b_n^l(x_{n-1}, \dots, x_{n-5}), n = 6, \dots, 1000)$$

in the second sample, i.e. the vector y, by solving the matrix equation in Proposition 2.3.5, replacing the exact ACVF γ with the sample ACVF you computed in Exercise 2. Evaluate the performance of your estimators by computing

$$\frac{1}{995} \sum_{n=6}^{1000} \left(b_n^l(x_{n-1}, \dots, x_{n-5}) - x_n\right)^2.$$

5. (3 points) Repeat the previous exercise using the Durbin–Levinson algorithm (Method 2.3.10) and compare your result to that of the previous exercise.

Deadline: Sunday, April 23 at 23.59 with bonus points and Sunday, May 28 at 23.59 without bonus points.
Requirement: *You must implement the algorithms from scratch.* This means that you may *only* use the most basic functions in MATLAB, e.g., vector/matrix addition and functions such as *exp*, *abs*, *sin* and *sum*. The prohibition on using other MATLAB functions extends to, e.g., the *mean* function which you should implement on your own. To pass the project, you need to score at least 50%, i.e., 8 out of a total of 16 points. Each additional point you score after that will give you half a bonus point on the exam.
Formalities: You are strongly encouraged to work in pairs, but everybody is responsible for their own project being handed in on time. Send your project report as one pdf document to both *annika.lang@chalmers.se* and *annika.lang.chalmers@analys.urkund.se*. Your report should include all plots, explanations, and answers to the questions as well as your implemented code in an appendix. You should try to write your report in LaTeX. If you do not, it is acceptable to scan your handwritten solutions to the theoretical parts of the project and include them in the pdf file. Put the original in the folder in front of *MVL2086* no later than *Monday, April 24, 10:00* or *Monday, May 29, 10:00*, respectively. The code should include comments to be readable. Emails received before the first deadline will be eligible for bonus points and should you not pass you will have a chance to correct it before the second deadline. Emails received after the first deadline will not be corrected until after the second deadline, but you are free to update your solution on your own and hand in another version before the second deadline. If you do not pass the project by the second deadline you may not hand in another version until the re-exam period.