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Financial Time Series TMS087/MSA410 – LP4 2016/17

Project 2

1. (3 points) Assume that the time series $Y = (Y_t, t \in \mathbb{N}_0)$ is given by the recursion

$$Y_t = \tfrac{1}{2} Y_{t-1} + \tilde{Z}_t,$$

where $\tilde{Z} = (\tilde{Z}_t, t \in \mathbb{N})$ is iid noise with $\tilde{Z}_t \sim \mathcal{N}(0, 3/4)$ for all $t \in \mathbb{N}$ and $Y_0 \sim \mathcal{N}(0,1)$ is independent of $\tilde{Z}$.

a) Derive the mean and the ACVF theoretically assuming that you know that Y is stationary.

b) Simulate $(Y_1, \dots, Y_{1000})$ with the recursion by setting $Y_0 \sim \mathcal{N}(0,1)$ and generating $(\tilde{Z}_1, \dots, \tilde{Z}_{1000})$ with `sqrt(3/4).*randn(1,1000)`.

c) Compute the sample mean and the sample ACVF with your implementations from Project 1 or the internal matlab functions. How well do they approximate your theoretical findings in a)?

2. (3 points) Let the time series $X = (X_t, t \in \mathbb{N})$ be given by the recursion

$$X_t = Y_t (Z_t + Z_{t-1}),$$

where Y is given in Problem 1. and $Z = (Z_t, t \in \mathbb{N}_0)$ is iid noise independent of Y with $Z_t \sim \mathcal{N}(0, 1/2)$ for all $t \in \mathbb{N}_0$).

a) Derive the mean and the ACVF theoretically and show that X is stationary.

b) Simulate $(X_1, \dots, X_{1000})$ with the recursion by generating Y is as in Problem 1. and $(Z_0, \dots, Z_{1000})$ with `sqrt(1/2).*randn(1,1001)`.

c) Compute the sample mean and the sample ACVF with your implementations from Project 1 or the internal matlab functions. How well do they approximate your theoretical findings in a)?

Please turn!

3. (4 points) It is shown in the lecture notes that X is an $\mathrm{MA}(1)$ process, i.e., there exist parameters θ and σ_V^2 and a white noise process $V \sim \mathrm{WN}(0, \sigma_V^2)$ such that $\tilde{X} = (\tilde{X}_t, t \in \mathbb{N})$ given by

$$\tilde{X}_t = V_t + \theta V_{t-1},$$

has the same mean and ACVF as X.

a) Derive theoretically the parameters θ and σ_V^2 for the given situation.

b) Simulate $(\tilde{X}_1, \ldots, \tilde{X}_{1000})$ with the computed parameters, where you can assume that $V_t \sim \mathcal{N}(0, \sigma_V^2)$.

c) Compute the sample mean and the sample ACVF with your implementations from Project 1 or the internal matlab functions. How well do they approximate the parameters from 2.a)?

d) The mean and the ACVF are two quantities to describe a stochastic process but they do not necessarily characterize it completely. Design a method to decide if your samples of X and $\tilde{X}$ follow the same distribution. You are allowed to generate as many samples of X and $\tilde{X}$ as you need and to use any internal matlab function you like. (*Hint: This is an open question that we recommend to work on once all other problems are solved.*)

4. (3 points) The goal of this problem is to use the generated samples X and $\tilde{X}$ from Problems 2. and 3. to recover the parameters θ and σ_V^2 of the $\mathrm{MA}(1)$ process. This should be done in the following manner:

a) Implement Method 3.2.19 on innovations estimation for an $\mathrm{MA}(1)$ model and compute estimators for the parameters θ and σ_V^2 using once X generated in Problem 2. and then $\tilde{X}$ generated in Problem 3.. What do you observe?

b) Repeat a) at least $M := 1000$ times and compute for all $m \leq M$

$$m^{-1} \sum_{i=1}^{m} |\hat{\theta}^{(i)} - \theta|^2 \qquad \text{and} \qquad m^{-1} \sum_{i=1}^{m} |\hat{\sigma}_V^{2(i)} - \sigma_V^2|,$$

where $\hat{\theta}^{(i)}$ and $\hat{\sigma}_V^{2(i)}$ denote the estimated parameter for the i-th sample of X. Plot your result as a function of m, i.e., with $1 \leq m \leq M$ on the x-axis.

c) Repeat b) with $\tilde{X}$ instead of X. Compare the results with those of b), e.g., by plotting them in the same figure.

See next page!

5. (3 points) The file *dow_jones.mat* contains 464 consecutive observations $(P_t, t = 0, \ldots, 463)$ of the closing daily values of the Dow-Jones Industrial Index. Your job is to make a simple analysis of the squared log-returns of this sample, something that is closely related to ARCH/GARCH processes that we consider at the end of the course. This is a fairly open exercise and you are meant to reason about your results with help of the course literature. If you want to consider additional models than the $\mathrm{MA}(1)$ process from the previous tasks, you are welcome to do so but make sure to solve the tasks below in addition to this.

a) Create a percentage log-return series sample $(R_t, t = 1, \ldots, 463)$ by computing

$$R_t := 100\,(\log(P_t) - \log(P_{t-1}))$$

and plot the series.

b) Consider the hypothesis $R \sim \mathrm{WN}(\mu_R, \sigma_R^2)$ for some $\mu_R \in \mathbb{R}$ and $\sigma_R^2 > 0$. Plot the sample ACF and PACF for R. Are these consistent with the hypothesis? Motivate your answer. For the PACF you may use the code from the lecture notes or internal matlab functions.

c) Plot the sample ACF and PACF for $(S_t, t = 1, \ldots, 463)$ where for $t = 1, \ldots, 463$, $S_t := R_t^2 - \mathbb{E}(R_t^2)$. Is it reasonable to assume that $S \sim \mathrm{WN}(0, \sigma_S^2)$? Is it reasonable to assume that $R \sim \mathrm{IID}(\mu_R, \sigma_R^2)$? Motivate your answer.

d) Consider the hypothesis that $(S_t, t = 1, \ldots, 463)$ is an $\mathrm{MA}(1)$ process. Are its sample ACF and PACF consistent with this hypothesis? What about an $\mathrm{MA}(2)$ process or an $\mathrm{AR}(p)$ process for some order $p \in \mathbb{N}$? Motivate your answer.

e) Use your code from Problem 4. to fit an $\mathrm{MA}(1)$ process to $(S_t, t = 1, \ldots, 463)$. Compute the *residuals* of your fit

$$e_t := S_t - b_t^l(S_1, \ldots, S_{t-1})$$

for $t = 1, \ldots, 463$, *where you are supposed to use your estimated model when computing* $b_t^l(S_1, \ldots, S_{t-1})$. Note that the computation of the predictions is particularly simple since you are predicting an $\mathrm{MA}(1)$ process (see Example 2.5.5 in [BD]). Plot the residuals e and their sample ACF. What do you observe? Do you think the model is appropriate for the data? Motivate your answer.

Deadline: Sunday, May 14 at 23.59 with bonus points and Sunday, May 28 at 23.59 without bonus points.
Requirement: *You must implement the algorithms from scratch if no other instructions are given in the task.* To pass the project, you need to score at least 50%, i.e., 8 out of a total of 16 points. Each additional point you score after that will give you half a bonus

point on the exam.

Formalities: You are strongly encouraged to work in pairs, but everybody is responsible for their own project being handed in on time. Send your project report as one pdf document to both *annika.lang@chalmers.se* and *annika.lang.chalmers@analys.urkund.se*. Your report should include all plots, explanations, and answers to the questions as well as your implemented code in an appendix. You should try to write your report in LaTeX. If you do not, it is acceptable to scan your handwritten solutions to the theoretical parts of the project and include them in the pdf file. Put the original in the folder in front of *MVL2086* no later than *Monday, May 15, 10:00* or *Monday, May 29, 10:00*, respectively. The code should include comments to be readable. Emails received before the first deadline will be eligible for bonus points and should you not pass you will have a chance to correct it before the second deadline. Emails received after the first deadline will not be corrected until after the second deadline, but you are free to update your solution on your own and hand in another version before the second deadline. If you do not pass the project by the second deadline you may not hand in another version until the re-exam period.