Nonlinear model exercises

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Ex. 1 — Let the monthly log returns $(r_t, t \in \mathbb{N})$, in percentages, of a stock follow the following Markov switching autoregressive model:

- $r_t := 2.5 + a_t$,
- $a_t := \sigma_t \epsilon_t$, where $(\epsilon_t, t \in \mathbb{N})$ is a series of independent, standard normally distributed random variables,
- $\sigma_t^2 := \begin{cases} 0.1a_{t-1}^2 + 0.93\sigma_{t-1}^2 & \text{if } S_t = 1, \\ 4.24 + 0.1a_{t-1}^2 + 0.78\sigma_{t-1}^2 & \text{if } S_t = 2, \end{cases}$

  where the transition probabilities are

$$P(S_t = 2|S_{t-1} = 1) = 0.3, \qquad P(S_t = 1|S_{t-1} = 2) = 0.25.$$

a) Assume that $a_{100} = 5$, $\sigma_{100}^2 = 10$, and $S_{100} = 2$ in the given example are known. Compute the one-step ahead volatility forecast, i.e. compute $\mathbb{E}[\sigma_{101}^2|S_{100} = 2]$.

b) Assume that a_{100} and σ_{100}^2 are as in the previous exercise and suppose you only know that $S_{100} = 2$ with 75% certainty. Compute the one-step ahead volatility forecast under this assumption. Hint: Use the fact that for any random variable X and any events A, B such that $P(A \cap B) = 0$ and $P(A \cup B) = 1$, $\mathbb{E}[X] = \mathbb{E}[X|A]P(A) + \mathbb{E}[X|B]P(B)$

Ex. 2 — Suppose that you have a data set with $n = 1000$ observations, and that you have fitted an ARMA(2,3) model to the data. You compute the residuals $(Z_t^2, t = 1, \ldots, 1000)$ and from these you compute the sample autocovariance function $\hat{\gamma}_{Z_t^2}$ for lags $h = 0, \ldots, 10$ to be

$$(\hat{\gamma}_{Z_t^2}(h))_{h=0}^{h=10} = (8.40, 0.24, -0.03, 0.14, -0.08, 0.16, -0.13, -0.20, -0.45, 0.05, 0.57).$$

Perform a statistical test with significance level $\alpha = 0.05$ to answer the question of whether this linear ARMA model is adequate given the data. *Hint: Use your favourite basic statistics text book to find critical values for the χ^2-distribution. Note that a one-sided test is appropriate in this case.*