Addition theorems for some distributions

X_1 and X_2 are independent random variables.

X_1	X_2	X_1+X_2
$B(n_1, p)$	$B(n_2, p)$	$B(n_1+n_2, p)$
$P(\lambda_1)$	$P(\lambda_2)$	$P(\lambda_1+\lambda_2)$
$NB(r_1, p)$	$NB(r_2, p)$	$NB(r_1+r_2, p)$
$E(\lambda)$	$E(\lambda)$	$\Gamma(2, \lambda)$
$\Gamma(n_1, \lambda)$	$\Gamma(n_2, \lambda)$	$\Gamma(n_1+n_2, \lambda)$
$N(\mu_1, \sigma_1)$	$N(\mu_2, \sigma_2)$	$N(\mu_1+\mu_2, \sqrt{\sigma_1^2+\sigma_2^2})$
$\chi^2(r_1)$	$\chi^2(r_2)$	$\chi^2(r_1+r_2)$
$C(a)$	$C(a)$	$C(2a)$

17.2 Probability Distributions

Discrete probability distributions (random variables)

Distribution	$P(X=x)$	Expectation μ	Variance σ^2	Example of application
Binomial $B(n, p)$	$\binom{n}{x}p^x(1-p)^{n-x}$ $x=0, 1, \ldots, n$	np	$np(1-p)$	The frequency in n independent trials has a binomial distribution. Probability in each trial$=p$
Geometric $G(p)$	$(1-p)^{x-1}p$ $x=1, 2, 3, \ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	The number of required trials until an event with probability p occurs has a geometric distribution
Poisson $P(\lambda)$	$e^{-\lambda}\lambda^x/x!$ $x=0, 1, 2, \ldots$	λ	λ	Distribution of number of points in random point process under certain simple assumptions. Approximation to the binomial distribution when n is large and p is small, $\lambda=np$.
Hypergeometric $H(N, n, p)$	$\frac{\binom{Np}{x}\binom{N-Np}{n-x}}{\binom{N}{n}}$	np	$np(1-p)\frac{N-n}{N-1}$	This distribution is used in connection with sampling without replacement from a finite population with elements of two different kinds
Negative binomial or Pascal $NB(r, p)$	$\binom{x-1}{r-1}p^r(1-p)^{x-r}$ $x=r, r+1, \ldots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	The number of required trials until an event with probability p occurs for the r^{th} time has a negative binomial distribution

Continuous probability distributions (random variables)

Distribution	$f(x)$	Expectation μ	Variance σ^2	Example of application
Uniform $U(a, b)$	$\frac{1}{b-a}$ $a\le x\le b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	Certain waiting times Rounding off errors
Exponential $E(\lambda)$	$\lambda e^{-\lambda x}$ $x\ge 0$	$1/\lambda$	$1/\lambda^2$	Distribution of length of life when no aging
Normed normal distribution $N(0, 1)$	$\varphi(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$	0	1	If X has a general normal distribution, then $(X-\mu)/\sigma$ has a normed normal distribution
General normal distribution $N(\mu, \sigma)$	$\frac{1}{\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right)$	μ	σ^2	Under general conditions, the sum of a large number of random variables is approximately normally distributed *(the central limit theorem)*
Gamma $\Gamma(n, \lambda)$	$\frac{\lambda^n}{\Gamma(n)}x^{n-1}e^{-\lambda x}$	$\frac{n}{\lambda}$	$\frac{n}{\lambda^2}$	Distribution of the sum of n independent random variables with an exponential distribution with parameter λ
χ^2 $\chi^2(r)$	$\frac{1}{2^{r/2}\Gamma(\frac{r}{2})}x^{\frac{r}{2}-1}e^{-\frac{x}{2}}$ $x\ge 0$ The parameter r is called the "number of degrees of freedom"	r	$2r$	Distribution of $u_1^2+u_2^2+\ldots+u_r^2$, where $u_1, u_2, \ldots, u_r$ are independent and have a normed normal distribution
t $t(r)$	$\frac{a_r}{b_r}\left(1+\frac{x^2}{r}\right)^{-\frac{r+1}{2}}$, $a_r=\Gamma\left(\frac{r+1}{2}\right)$ $b_r=\sqrt{r\pi}\,\Gamma\left(\frac{r}{2}\right)$	$0, r>1$	$\frac{r}{r-2}, r>2$	Distribution of $u/\sqrt{X/r}$ where u and X are independent, u has a normed normal distribution and X a χ^2-distribution with r degrees of freedom
F $F(r_1, r_2)$	$\frac{a_r x^{(r_1/2)-1}}{b_r(r_2+r_1x)^{\frac{r_1+r_2}{2}}}$ $x\ge 0$ $a_r=\Gamma(\frac{r_1+r_2}{2})\,r_1^{r_1/2}r_2^{r_2/2}$ $b_r=\Gamma(\frac{r_1}{2})\,\Gamma(\frac{r_2}{2})$	$\frac{r_2}{r_2-2}$ $r_2>2$	$\frac{2r_2^2(r_1+r_2-2)}{r_1(r_2-2)^2(r_2-4)}$ $r_2>4$	Distribution of $(X_1/r_1)/(X_2/r_2)$, where X_1 and X_2 are independent and have χ^2-distributions with r_1 and r_2 degrees of freedom