Formelsamling för tenta i Dataanalys och Statistik, tms136, I2.

Combinatorics:

$\binom{n}{r} = \frac{n!}{r!(n-r)!}$ = number of subsets of size r from a set of size n

$n! = n(n-1)(n-2)\cdot \ldots \cdot 2 \cdot 1$ = number of permutations of n different objects.

Events

$P(B) = P(B\cap A) + P(B\cap A') = P(B|A)P(A) + P(B|A')P(A')$

$P(A\cap B) = P(B|A)P(A) = P(A|B)P(B)$

$P(A\cup B) = P(A) + P(B) - P(A\cap B) \le 1$

$E_1 \cap E_2 = E_1 \cap E_1' = 0$ – Mutually exclusive

Probability mass/density function:

(1) $f(x) \ge 0$

(2) $\sum_{i=1}^{n} f(x_i) = \int_{-\infty}^{+\infty} f(x)dx = 1$

(3) $f(x_i) = P(X = x_i)$, for discrete variables

(4) $P(a \le X \le b) = \int_a^b f(x)dx$, for continuous variables

Cumulative distribution function:

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)du, \qquad for -\infty < x < \infty$$

Mean and variance of a random variable:

$$\mu = E(X) = \sum_x xf(x) = \int_{-\infty}^{\infty} xf(x)dx$$

$$\sigma^2 = Var(X) = \sum_x x^2 f(x) - \mu^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2 = E(X^2) - (E(X))^2$$

Expected value of a function of a continuous random value:

$$E[h(x)] = \int_{-\infty}^{\infty} h(x)f(x)dx$$

Discrete uniform distribution, $Uni(\{x_1, \ldots, x_n\})$

$$f(x_i) = \frac{1}{n}, for\ all\ x_i$$

Continuous uniform distribution, $Uni([a,b])$

$$f(x) = \frac{1}{(b-a)}, E(X) = \frac{(a+b)}{2}, Var(X) = \frac{(b-a)^2}{12}$$

Binomial distribution, $Bin(n,p)$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$E(X) = np, \qquad Var(x) = np(1-p)$$

Geometric distribution, $Geo(p)$

$$f(x) = (1-p)^{x-1}p, \qquad x = 1,2,\ldots$$

$$\mu = E(X) = \frac{1}{p}, \qquad \sigma^2 = Var(x) = (1-p)/p^2$$

Poisson distribution, $Poi(\lambda)$

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots$$

$$\mu = E(X) = Var(X) = \lambda$$

Normal distribution, $N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$E(X) = \mu, \qquad Var(X) = \sigma^2$$

Exponential distribution, $Exp(\lambda)$:

$f(x) = \lambda e^{-\lambda x}$, for $0 \le x \le \infty$

$\mu = E(X) = 1/\lambda$, $\sigma^2 = Var(X) = 1/\lambda^2$

Normal approximation to the binomial distribution:

$$Z = \frac{X - np}{\sqrt{np(1-p)}}$$

is approximately a standard normal random variable.

The approximation is good for $np > 5$ and $n(1-p) > 5$

Normal approximation to the Poisson distribution:

$Z = \frac{X-\lambda}{\sqrt{\lambda}}$ is good for $\lambda > 5$

Joint probability mass function of two random variables:

(1) $f_{XY}(x,y) \ge 0$

(2) $\sum_x \sum_y f_{XY}(x,y) = \iint_{-\infty}^{\infty} f_{XY}(x,y)dxdy = 1$

(3) $f_{XY}(x,y) = P(X = x, Y = y)$ (discrete case)

(4) $P(X < x, Y < y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{XY}(x,y)dydx$ (continuous case)

Marginal probability mass function:

$f_X(x) = P(X = x) = \sum_y f_{XY}(x,y)$ (discrete case)

$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)dy$ (continuous case)

Conditional probability mass/density function of Y given X=x is**:**

$$f_{Y|x}(y) = \frac{f_{XY}(x,y)}{f_X(x)}, \qquad for\ f_X(x) > 0$$

Covarience:

$$Cov(X,Y) = \sigma_{XY} = E[(X-\mu_X)(Y-\mu_Y)] = E(XY) - \mu_X\mu_Y$$

Correlation:

$$\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y}, \quad -1 \le \rho_{XY} \le 1$$

If X & Y are independent random variables, then $Cov(X,Y) = \rho_{XY} = 0$ ($\not\Leftarrow$)

The **standard error** of an estimator $\widehat{\Theta}$ is its standard deviation, given by:

$$\sigma_{\widehat{\Theta}} = \sqrt{V(\widehat{\Theta})}$$

The **mean squared error** of the estimator $\widehat{\Theta}$ of the parameter θ is:

$MSE(\widehat{\Theta}) = E(\widehat{\Theta} - \theta)^2$

An estimator $\widehat{\Theta}$ is called **unbiased** if

$E(\hat{\theta}) - \theta = 0$

Likelihood function:

$L(\theta) = \prod_{i=1}^{n} f(x_i|\theta) = f(x_1|\theta)f(x_2|\theta)\cdots f(x_n|\theta)$

If $x_1, \ldots, x_n$ is a sample of n observations, the **sample variance** is:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{n} {x_i}^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}{n-1}$$

Confidence interval on the mean, variance known:

$$\bar{x} - z_{\alpha/2}\sigma/\sqrt{n} \le \mu \le \bar{x} + z_{\alpha/2}\sigma/\sqrt{n}$$

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}, \qquad Choice\ of\ n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2, E = |\bar{x} - \mu|$$

Confidence interval on the mean, variance unknown:

$$\bar{x} - t_{\alpha/2,n-1}s/\sqrt{n} \le \mu \le \bar{x} + t_{\alpha/2,n-1}s/\sqrt{n}$$

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

Random sample normal disr. mean=μ, var=σ^2, S^2=sample var.

$$\chi^2_{n-1} = \frac{(n-1)S^2}{\sigma^2}\ has\ \chi^2\ dist.with\ n-1\ degrees\ of\ freedom$$

CI on variance, s^2=sample variance, σ^2 unknown

$$\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2},n-1}} \le \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2},n-1}}$$

Lower and upper confidence bounds on σ^2:

$$\frac{(n-1)s^2}{\chi^2_{\alpha,n-1}} \le \sigma^2\ and\ \sigma^2 \le \frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}}$$

Proportion:

If *n* is large, the distribution of

$$Z = \frac{X - np}{\sqrt{np(1-p)}} = \frac{\hat{P} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

is approximately standard normal.

CI on proportion (obs, lower, upper change $z_{\alpha/2}$ to z_{α})**:**

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Sample size for a specified error on binomial proportion:

$$n = \left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 p(1-p), n\ is\ \max for\ p = 0.5$$

CI, **difference in mean**, variances known:

$$\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

for one-sided, change $z_{\alpha/2}$ to z_{α}.

Sample size for a CI on difference in mean, variances known:

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 (\sigma_1^2 + \sigma_2^2)$$

CI Case 1, **difference in mean**, variance unknown & equal:

$$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2,n_1+n_2-2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{\alpha/2,n_1+n_2-2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}$$

CI Case 2, **difference in mean**, variance unknown, not equal:

$$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

v is degrees of freedom for $t_{\alpha/2}$,if not integer, round down.

CI for μ_D from paired samples:

$$\bar{d} - t_{\alpha/2,n-1}s_D/\sqrt{n} \le \mu_D \le \bar{d} + t_{\alpha/2,n-1}s_D/\sqrt{n}$$

Approximate CI on difference in population proportions:

$$\hat{p}_1 - \hat{p}_2 - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}} \le p_1 - p_2$$
$$\le \hat{p}_1 - \hat{p}_2 + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Hypothesis test:

1. Choose parameter of interest
2. H_0:
3. H_1:
4. α=
5. The test statistic is
6. Reject H_0 at α= … if
7. Computations
8. Conclusions

Test on mean, variance known

$$H_0: \mu = \mu_0, \qquad Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

Alternative hypothesis	Rejection criteria
$H_1: \mu \ne \mu_0$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
$H_1: \mu > \mu_0$	$z_0 > z_\alpha$
$H_1: \mu < \mu_0$	$z_0 < -z_\alpha$

Test on mean, variance unknown

$$H_0: \mu = \mu_0, \qquad T_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

Alternative hypothesis	Rejection criteria
$H_1: \mu \ne \mu_0$	$t_0 > t_{\alpha/2,n-1}$ or $t_0 < -t_{\alpha/2,n-1}$
$H_1: \mu > \mu_0$	$t_0 > t_{\alpha,n-1}$
$H_1: \mu < \mu_0$	$t_0 < -t_{\alpha,n-1}$

Test in the variance of a normal distribution:

$$H_0: \sigma^2 = \sigma_0^2, \qquad \chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

Alternative hypothesis	Rejection criteria
$H_1: \sigma^2 \ne \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$
$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha,n-1}$
$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$

Approximate test on a proportion:

$$H_0: p = p_0, \qquad Z_0 = \frac{X - np_0}{\sqrt{np_0(1-p_0)}}$$

Alternative hypothesis	Rejection criteria (**)
$H_1: p \ne p_0$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
$H_1: p > p_0$	$z_0 > z_\alpha$
$H_1: p < p_0$	$z_0 < -z_\alpha$

App. Sample size for a 2-sided test on a proportion:

$$n = \left[\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} + z_\beta\sqrt{p(1-p)}}{p - p_0}\right]^2, for\ 1 - sided\ use\ z_\alpha$$

Test on the differens in mean, variance known

$$H_0: \mu_1 - \mu_2 = \Delta_0, \qquad Z_0 = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Alternative hypothesis	Rejection criteria
$H_1: \mu_1 - \mu_2 \ne \Delta_0$	$z_0 > z_{\alpha/2}$ or $z_0 < -z_{\alpha/2}$
$H_1: \mu_1 - \mu_2 > \Delta_0$	$z_0 > z_\alpha$
$H_1: \mu_1 - \mu_2 < \Delta_0$	$z_0 < -z_\alpha$

Sample size, 1-sided test on difference in mean, with power of at least 1-β, $n_1 = n_2 = n$, variance known:

$$n = \frac{(z_\alpha + z_\beta)^2(\sigma_1^2 + \sigma_2^2)}{(\Delta - \Delta_0)^2}$$

Tests on diff. in mean, variances unknown and equal:

$$H_0: \mu_1 - \mu_2 = \Delta_0, \qquad T_0 = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Alternative hypothesis	Rejection criteria
$H_1: \mu_1 - \mu_2 \ne \Delta_0$	$t_0 > t_{\alpha/2,n_1+n_2-2}$ or $t_0 < -t_{\alpha/2,n_1+n_2-2}$
$H_1: \mu_1 - \mu_2 > \Delta_0$	$t_0 > t_{\alpha,n_1+n_2-2}$
$H_1: \mu_1 - \mu_2 < \Delta_0$	$t_0 < -t_{\alpha,n_1+n_2-2}$

Tests on diff. in mean, variances unknown and not equal:

$$\text{If } H_0: \mu_1 - \mu_2 = \Delta_0 \text{ is true, the statistic } T_0 = \frac{\bar{X}_1 - \bar{X}_2 - \Delta_0}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

is distributed app. as t with ν degrees of freedom, $\sim t(\nu)$

Paired *t*-test:

$$H_0: \mu_D = \mu_1 - \mu_2 = \Delta_0, \qquad T_0 = \frac{\bar{D} - \Delta_0}{S_D/\sqrt{n}}, \qquad d = \frac{\mu_D}{\sigma_D} = \frac{\mu_1 - \mu_2}{\sqrt{\sigma_1^2 + \sigma_2^2}}$$

Alternative hypothesis	Rejection criteria
$H_1: \mu_D \ne \Delta_0$	$t_0 > t_{\alpha/2,n-1}$ or $t_0 < -t_{\alpha/2,n-1}$
$H_1: \mu_D > \Delta_0$	$t_0 > t_{\alpha,n-1}$
$H_1: \mu_D < \Delta_0$	$t_0 < -t_{\alpha,n-1}$

Approximate tests on the difference of two population proportions:

$$H_0: p_1 = p_2,\ Z_0 = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}(1-\hat{P})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}, \hat{P} = \frac{X_1 + X_2}{n_1 + n_2}, see\ (**) \uparrow$$

Goodness of fit:

$$X_0^2 = \sum_{i=1}^{k}\frac{(O_i - E_i)^2}{E_i} > \chi^2_{\alpha,k-p-1}$$

Expected frequency: $E_i = np_i,\ p_i = P(X = x) = f(x)$

The power of a test: $= 1 - \beta$

$$\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)$$

The P-value is the smallest level of significance that would lead to rejection of the null hypothesis H_0 with the given data.

$$P = \begin{cases} 2[1 - \Phi(|z_0|)]\ for\ a\ two - tailed\ test: & H_o: \mu = \mu_0 \quad H_1: \mu \ne \mu_0 \\ 1 - \Phi(z_0)\ for\ a\ upper - tailed\ test: & H_o: \mu = \mu_0 \quad H_1: \mu > \mu_0 \\ \Phi(z_0)\ for\ a\ lower - tailed\ test: & H_o: \mu = \mu_0 \quad H_1: \mu < \mu_0 \end{cases}$$

Fitted or estimated regression line: $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}, \qquad \bar{y} = \left(\frac{1}{n}\right)\sum_{i=1}^{n} y_i, \qquad \bar{x} = \left(\frac{1}{n}\right)\sum_{i=1}^{n} x_i, SS_T = SS_R + SS_E$$

$$\hat{\beta}_1 = \frac{S_{xy}}{Sxx} = \frac{\sum_{i=1}^{n} y_i x_i - \frac{(\sum_{i=1}^{n} y_i)(\sum_{i=1}^{n} x_i)}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}, e_i = y_i - \hat{y}_i,\ SS_E = \sum_{i=1}^{n} e_i^2$$

$$\hat{\sigma}^2 = \frac{SS_E}{n-2}, SS_E = SS_T - \hat{\beta}_1 S_{xy}, SS_T = \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - n\bar{y}^2$$

Ci: $\beta_1 \in \left(\hat{\beta}_1 \pm t_{\alpha/2,n-2}\sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}\right), \beta_0 \in \left(\hat{\beta}_0 \pm t_{\alpha/2,n-2}\sqrt{\hat{\sigma}^2\left[\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right]}\right)$

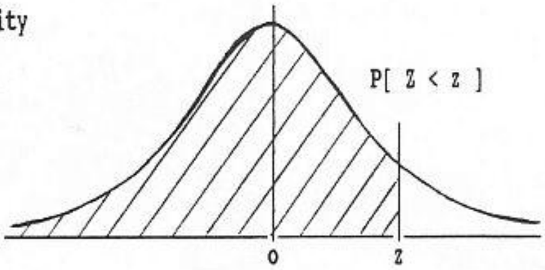
$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T}, SS_R = \hat{\beta}_1 S_{xy}, F_0 = \frac{\frac{SS_R}{1}}{\frac{SS_E}{n-2}} = \frac{MS_E}{MS_E}$$

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z
i.e.

$$P[\ Z < z\ ] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\ dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9874	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9924	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

z	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
P	0.9986	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000