Lab 2

Aim: Create optimal portfolio

Data: Price for 7 stocks over time

Actions: How much money to invest in each stock (the portfolio)

Action described as vector $w = [w_1, \ldots, w_7]^T$ (column) such that $\sum_{i=1}^{7} w_i = 1$

w_i is the part of money invested in stock i

States: Change in price for each stock over some future period.

State described as random vector
$X = [X_1, \ldots, X_7]^T$

X_i is the (unknown) log-return for stock i.

We will assume $X \sim N(\mu, \Sigma)$ (vector, matrix), the joint multivariate normal distribution. (In science, we sometimes make simplifying assumptions that are only approx. correct.)

Then the profit is $Y = w^T X = \sum_{i=1}^{7} w_i X_i$.

Problem: Find w that maximizes expected utility $U(w)$

Subject to $w_1, \ldots, w_7 \geq 0$, $\sum_{i=1}^{7} w_i = 1$

Log-returns

Let $S(t)$ be the price of a stock at time t

$\{S(0), S(1), \ldots\}$ is a non-stationary stochastic process: the distribution of $S(t)$ changes with t.

The returns $\frac{S(t) - S(t-1)}{S(t-1)}$ are approx. stationary.

The log-returns $X(t) = \log \frac{S(t)}{S(t-1)}$ are

approx. normally distributed, indep. and form an approx. stationary process.

(You will check assumptions in the lab)

Autocorrelation function (ACF)

ACF measures correlation between values of the process and values time lag h later:

$$r_X(h) = \frac{Cov(X(t), X(t+h))}{Var(X(t))}$$

If log-returns are indep. in time, then $r_X(h) = 0$ for all h.

Estimate $r_X(h)$ and check if significantly different from 0.

(See details in lab intro

Multivariate (joint) distributions

X, Y continuous r.v.'s

Covariance matrix $\Sigma = \begin{bmatrix} Var(X) & Cov(X,Y) \\ Cov(X,Y) & Var(Y) \end{bmatrix}$

Correlation $\rho = \dfrac{Cov(X,Y)}{\sqrt{Var(X)\,Var(Y)}} \Rightarrow -1 \leq \rho \leq 1$

(Correlation is normalized so easier to interpret than covariance

The multivariate normal distribution

$\mathbb{X} = \{X_1, \dots, X_n\}$, $X_i \sim N(\mu_i, \sigma_i^2)$
(so not identically dist., not indep)

$\mathbb{X} \sim N(\mu, \Sigma)$, $\mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_n \end{pmatrix}$,

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \cdots \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \\ & & \ddots & \\ \vdots & & & \sigma_n^2 \end{bmatrix},$$ $n \times n$ positive definite covariance matrix

where $\sigma_{ij} = \mathrm{Cov}(X_i, X_j)$

$$f(x) = \frac{1}{\sqrt{(2\pi)^n |\Sigma|}} e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)}$$

where $|\Sigma|$ is the determinant of Σ.

Expected utility

Utility for profit y: $u(y) = 1 - e^{-ky}$

Profit $Y = w^T X$, $X \sim N(\mu, \Sigma) \Rightarrow Y \sim N(w^T\mu, w^T \Sigma w)$

Expected utility: $U(w) = E_Y[u(Y)] =$

$$= \int u(y)\, f(y)\, dy = \int (1 - e^{-ky}) \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(y - w^T\mu)^2}{2\sigma^2}}$$

where $\sigma^2 = w^T \Sigma w$

Hints: Before integrating: plot $u(y)$ and $u(y)f(y)$ on $[-1, 1]$

- Use quad for integration
- Change tolerance for fminsearch