Exercise session 6, Stochastic Calculus Part I.

1 Consider the Ornstein-Uhlenbeck process, described by

$$dX_t = -\alpha X_t dt + \sigma dB_t, \quad \alpha > 0.$$

Find X_t and its distribution. What happens asymptotically as $t \to \infty$?

2 Consider the Black-Scholes process, described by

$$dX_t = X_t(rdt + \sigma dB_t).$$

What happens asymptotically to the solution X_t?

3 Solve the SDE

$$dX_t = \left[\alpha b(X_t)h(X_t) + \frac{1}{2}b(X_t)b'(X_t)\right]dt + b(X_t)dB_t,$$

for $t > 0$, $\alpha \in \mathbb{R}$ and $h'(x) = \frac{1}{b(x)}$.

4 Show that there exists a unique strong solution to the SDE

$$dX_t = [\ln(X_t^2 + 1) + X_t \sin t]dt + X_t \arctan t dB_t.$$