

## TMS 165/MSA350 Stochastic Calculus Part I Fall 2013

### Home exercises for Chapters 1-2 in Klebaner's book

**Task 1.** Find a function with infinite variation over a finite interval.

**Task 2.** Express  $[f, g]$  in terms of  $[f + g]$  and  $[f - g]$ .

**Task 3.** Explain why the Riemann-Stieltjes integral  $\int f dg$  defined through Equation 1.19 in Klebaner's book is not well-defined when  $[f, g] \neq 0$ .

**Task 4.** For the function  $f : [0, 1] \rightarrow \{0, 1\}$  given by  $f(x) = 0$  for  $x \in [0, 1] \setminus \mathbb{Q}$  and  $f(x) = 1$  for  $x \in [0, 1] \cap \mathbb{Q}$  we have  $\{x \in [0, 1] : f(x) = 1\} \subseteq \cup_{i=1}^{\infty} [q_i - 2^{-i}\varepsilon, q_i + 2^{-i}\varepsilon] \equiv L(\varepsilon)$  for each  $\varepsilon > 0$  and any enumeration  $\{q_i\}_{i=1}^{\infty}$  of  $[0, 1] \cap \mathbb{Q}$ , where the length of  $L(\varepsilon)$  is less or equal  $\varepsilon$ . Find the value of the Lebesgue integral  $\int_0^1 f(x) dx$ .

**Task 5.** Prove Equation 2.19 in Klebaner's book.

**Task 6.** Find  $\mathbf{E}\{X | \sigma(Y)\}$  for a standardized (to have zero mean and unit variance) bivariate normal distributed random variable  $(X, Y)$  such that  $X$  and  $Y$  has correlation  $\rho \in (-1, 1)$ .