

TMS 165/MSA350 Stochastic Calculus

Home Exercises for Chapters 1-2 in Klebaner's Book

Task 1. Find a function with infinite variation over a finite interval.

Task 2. Express $[f, g]$ in terms of $[f + g]$ and $[f - g]$.

Task 3. Explain why the Riemann-Stieltjes integral $\int f dg$ defined through Equation 1.19 in Klebaner's book is not well-defined when $[f, g] \neq 0$.

Task 4. For the function $f : [0, 1] \rightarrow \{0, 1\}$ given by $f(x) = 0$ for $x \in [0, 1] \setminus \mathbb{Q}$ and $f(x) = 1$ for $x \in [0, 1] \cap \mathbb{Q}$ we have $\{x \in [0, 1] : f(x) = 1\} \subseteq \cup_{i=1}^{\infty} [q_i - 2^{-i}\varepsilon, q_i + 2^{-i}\varepsilon] \equiv L(\varepsilon)$ for each $\varepsilon > 0$ and any enumeration $\{q_i\}_{i=1}^{\infty}$ of $[0, 1] \cap \mathbb{Q}$, where the length of $L(\varepsilon)$ is less or equal ε . Find the value of the Lebesgue integral $\int_0^1 f(x) dx$.

Task 5. Prove Equation 2.19 in Klebaner's book.

Task 6. Find $\mathbf{E}\{X | \sigma(Y)\}$ for a standardized (to have zero mean and unit variance) bivariate normal distributed random variable (X, Y) such that X and Y has correlation $\rho \in (-1, 1)$.