TMS 165/MSA350 Stochastic Calculus

Home Exercises for Chapters 1-2 in Klebaner's Book

Task 1. Find a function with infinite variation over a finite interval.

Task 2. Express $[f,g]$ in terms of $[f+g]$ and $[f-g]$.

Task 3. Explain why the Riemann-Stieltjes integral $\int f\,dg$ defined through Equation 1.19 in Klebaner's book is not well-defined when $[f,g]\neq 0$.

Task 4. For the function $f:[0,1]\to\{0,1\}$ given by $f(x)=0$ for $x\in[0,1]\setminus\mathbb{Q}$ and $f(x)=1$ for $x\in[0,1]\cap\mathbb{Q}$ we have $\{x\in[0,1]:f(x)=1\}\subseteq\cup_{i=1}^{\infty}[q_i-2^{-i}\varepsilon,q_i+2^{-i}\varepsilon]\equiv L(\varepsilon)$ for each $\varepsilon>0$ and any enumeration $\{q_i\}_{i=1}^{\infty}$ of $[0,1]\cap\mathbb{Q}$, where the length of $L(\varepsilon)$ is less or equal ε. Find the value of the Lebesgue integral $\int_0^1 f(x)\,dx$.

Task 5. Prove Equation 2.19 in Klebaner's book.

Task 6. Find $\mathbf{E}\{X\,|\,\sigma(Y)\}$ for a standardized (to have zero mean and unit variance) bivariate normal distributed random variable (X,Y) such that X and Y has correlation $\rho\in(-1,1)$.