Stochastic Calculus Part II Fall 2009

Hand in 3

1. Let $B(t)$ be a Brownian motion.
Find a process $\tau(t)$ such that $B(\tau(t)) = t$.
Motivate why (by which property of Brownian motion) $B(\tau(t))$ really equals t (almost surely). (2 points)

2. Invent your own exercise 7.13. That is let $M(t) = \int_0^t f(s)\,dB(s)$ for a suitable function f of your own choice (but do not choose $f(s)$ constant, $f(s) = s$ or $f(s) = e^s$) and then find a function g such that $M(g(t))$ is a Brownian motion.
Does f have to be injective?
Does g have to be non-decreacing?
Does g have to be increasing? (4 points)

3. Let $M(t) = \displaystyle\int_0^t \frac{1}{1+s^2}\,dB(s)$. Is it possible to find a function g such that $M(g(t))$ is a Brownian motion? (As usual you should motivate your answer!). (2 points)