

Stochastic Calculus Part II Fall 2009

Hand in 4

Let $\{B_t\}_{t \geq 0}$ be a Brownian motion and let $\{N_t\}_{t \geq 0}$ be a Poisson process with intensity 1 that is independent of B .

1 Let $T > 0$ be a constant time and define

$$M_t = \begin{cases} 0 & \text{for } t \leq T \\ B_{t-T} & \text{for } t > T \end{cases}.$$

Let the filtration be $\mathcal{F}_t = \mathcal{F}_t^M \vee \mathcal{F}_t^N = \sigma(M(s) : s \leq t) \vee \sigma(N(s) : s \leq t)$.

- (a) Show that $X_t = M_t + N_t$ is a semimartingale. (1 point)
- (b) Establish that N_{t-} is predictable. (1 point)
- (c) Show that $\int_0^t N_{s-} dX_s$ is well-defined for $t \geq 0$. (1 point)
- (d) What type of probability distribution does $\int_0^t N_{s-} dX_s$ have? (1 point)

2 Let $\alpha > 0$ and $\sigma > 0$ be constants. Consider the SDE $dX_t = -\alpha X_{t-} dt + \sigma dN_t$ (i.e., a non-Gaussian Langevin equation/Ornstein-Uhlenbeck process).

- (a) Find the solution to the equation. (1 point)
- (b) Make computer simulations to estimate $E(X_t)$ and $Var(X_t)$ for $\alpha = \sigma = 1$ and $X_0 = 0$. (1 point)

Let $\{X_t\}_{t \geq 0}$ and $\{Y_t\}_{t \geq 0}$ be semimartingales.

- 3(a) Motivate that $[X, Y]_t = \sum_{s \leq t} \Delta X_s \Delta Y_s$ when one of the processes X and Y has finite variation. (1 point)
- (b) Show that if one of the processes X and Y is continuous with finite variation then $[X, Y] = 0$. (1 point)