Answers to some problems recommended for work on one's own

Chapter 2.

29. a) e^{-10}; e^{-5}, b) $e^5 - e^{10}$.

50. $P\{ABC\} = P\{C\}P\{B|C\}P\{A|BC\}$.

57. a) $(p_1 + p_2)/2$; b) $p_1/(p_1 + p_2)$.

80. $(k-1)p^2(1-p)^{k-2}$.

Chapter 3.

12. 1/4

19. a) $c = 6$; b) $P\{1/2 \le X \le 3/4\} = 0.343$.

60. a) $f_Y(y) = 3/4(1 - \sqrt{Y\pi})$, $0 \le y \le \pi$.

60. b) $f_Y(y) = 2\Big(\dfrac{3}{4\pi}\Big)^{2/3} y^{-1/3}\Big[1 - \Big(\dfrac{3y}{4\pi}\Big)^{1/3}\Big]$, $0 \le y \le 4\pi/3$.

60. c) $f_Y(y) = (6/n)y^{\frac{2-n}{n}}\big(1 - y^{1/n}\big)$, $0 \le y \le 1$.

65. $E[X] = \dfrac{u+1}{2}$, $\sigma_x^2 = \dfrac{u^2-1}{12}$.

67. a) $E[X] = \alpha$, $E[X^2] = \alpha(\alpha+1)$.

67. b) $E[X] = \alpha = \lambda t = \text{arrival rate} \cdot \text{time}$.

82. $E[Y] = p$, $Var(Y) = 1 - p$, $P\{|Y - p| > a\} \le \dfrac{qp}{na^2} \to 0$, $n \to \infty$.

Chapter 4.

11. a) $k = 1/\pi$; $k = 1/2$; $k = 2$.

11. b) (i) $f_Y(y) = \dfrac{2\sqrt{1-y^2}}{\pi}$, $-1 < y < 1$, (ii) $f_Y(y) = 1 - |y|$, $-1 < y < 1$, (iii) $f_Y(y) = 2(1-y)$, $0 < y < 1$.

17. a) $P\{Y \le y|X = 1\}P\{X = 1\} = \begin{cases} \frac{1}{4}e^{\alpha(y-1)}, & \text{if } y < 1, \\ \frac{1}{4}(2 - e^{-\alpha(y-1)}), & \text{if } y < 1. \end{cases}$

17. b) $f_Y(y) = \frac{1}{2} f_N(y-1) + \frac{1}{2} f_N(y+1)$.

17. c) $X = 1$ is more likely.

24. a) $P\{X^2 < 1/2, \ |Y-1| < 1/2\} = \frac{1}{2\sqrt{2}}$.

24. b) $P\{X/2 < 1, \ Y > 0\} = 1$.

24. c) $P\{XY < 1/2\} = 0.85$.

24. d) $P\{\min(X,Y) > 1/3\} = 16/81$.

26. a) The first n trials are independent of the remaining m trials.

26. b) $p_n(N) = \binom{n}{N} p^N (1-p)^{n-N}, \quad p_m(M) = \binom{m}{M} p^M (1-p)^{m-M}$
$p_{n+m}(N+M) = p_n(N) p_m(M)$.

26. c) $p_{n+m}(k) = \binom{n+m}{k} p^k (1-p)^{n+m-k}$.

32 i) $f_Y(y|x) = \frac{1}{2\sqrt{1-x^2}}, \quad |x| < 1, \quad |y| < \sqrt{1-x^2}$.

32 ii) $f_Y(y|x) = \frac{1}{2-|x|}, \quad |x| < 1, \quad |y| < 1 - |x|$.

32 iii) $f_Y(y|x) = \frac{1}{1-x}, \quad 0 < x << 1, \quad 0 < y < 1 - x$.

39. a) $k = 2/3$, b) $f_{X,Y}(x,y) = \frac{2}{3}(x + y + 1/2)$; $\quad f_Z(z|x,y) = \frac{x+y+z}{x+y+1/2}$.

49. $f_Z(z) = \begin{cases} -\frac{\ln z}{2}, & 0 < z \leq 1, \\ -\frac{\ln(-z)}{2}, & -1 \leq < z < 0. \end{cases}$

60. $E[X - Y] = 1$.

64. i), ii): $\rho_{X,Y,} = 0$, iii)$\rho_{X,Y,} = -1$.

Chapter 5.

24. 0.996.

28. 0.025.

Chapter 6.

4. b) $x^{1/n}$.

4. c) $P\{X_n \leq x,\, X_{n+1} \leq y\} = \min(x^{1/n},\, y^{1/n})$.

4. d) $\frac{1}{n+1}$; $\quad \frac{1}{2n+k+1} - \frac{1}{n+1} \cdot \frac{1}{n+k+1}$.

5. a) $P\{X(t) = 1\} = P\{X(t) = -1\} = \frac{1}{2}, \quad t \in [0,\, 1]$.

5. b) $m_X(t) = 0$.

5. c)

$t \in [0,\, 1],\, t + d \in [0,\, 1]: \quad P\{X(t) = \pm 1,\; X(t+d) = \pm 1\} = \frac{1}{2}$.

$t \in [0,\, 1],\; t + d \notin [0,\, 1]: \quad P\{X(t) = \pm 1,\; X(t+d) = 0\} = \frac{1}{2}$.

$t \notin [0,\, 1],\; t + d \notin [0,\, 1]: \quad P\{X(t) = 0,\; X(t+d) = 0\} = 1$.

5. d)

$$C_X(t,\, t+d) = \begin{cases} 1, & t \in [0,\, 1],\, t+d \in [0,\, 1] \\ 0, & \text{otherwise.} \end{cases}$$

16. a) $E[X(t)] = 0, \quad C_X(t_1,\, t_2) = \sigma^2 \cos \omega (t_1 - t_2)$.

16. b) $f_{X(t),\, X(t+s)}(x_1,\, x_2) = \dfrac{\exp\left\{-\frac{x_1^2 - 2\cos \omega s x_1 x_2 + x_2^2}{2\sigma^2 sin^2 \omega s}\right\}}{2\pi\sigma^2 |\sin \omega s|}$.

19. $E[Y(t)] = C_X(t,\, t); \quad C_Y(t_1,\, t_2) = 2C_X^2(t_1,\, t_2)$.

25. a) $E[M_n] = E[X], \quad C_M(n,\, k) = \frac{\sigma_X^2}{nk} \min(n,\, k)$.

25. b) M_n does not have ind. inctements.

29. a) $M_n \sim N(0,\, 1/n)$.

29. b) $f_{M_n,\, M_{n+k}}(x,\, y) = n(n+k) \frac{e^{-n^2 x^2/2n}}{\sqrt{2\pi n}} \frac{e^{-[(n+k)y - nx]^2/2k}}{\sqrt{2\pi k}}$.

31. e^{-5}.

32. $P\{N(t) = k\} = \frac{(p\lambda t)^k}{k!} e^{-p\lambda t}$.

34. a) X_i= time till first arrival in line i.

$P\{X_1 < X_2\} = \frac{\lambda_1}{\lambda_1+\lambda_2}$.

34. b) $Z = \min(X_1, X_2) \sim \text{Exp}(\lambda_1 + \lambda_2)$.

34. c) $N(t)$ is Poisson RP with rate $\lambda_1 + \lambda_2$.

48. a) $f_{Z(t)}(z) = \frac{\exp\left\{-\frac{z^2}{2Var(Z(t))}\right\}}{\sqrt{2\pi Var(Z(t))}}$.

48. b) $m_Z(t) = 0$,

$C_Z(t_1, t_2) = \alpha(1 + a^2)\min(t_1, t_2) - a\alpha[\min(t_1 - s, t_2) + \min(t_1, t_2 - s)]$.

49. a) $m_Z(t) = 0$,

49. b) $f_{Z(t)}(z) = \frac{\exp\left\{-\frac{z^2}{\alpha(4\alpha t - 3\alpha s)}\right\}}{\sqrt{2\pi(4\alpha t - 3\alpha s)}}$.

54. a) yes; b) yes.

56. a) $Y(t)$ is WSS.

56. b) $Y(t)$ is a Gaussian process with mean $(1 - a)m_X$ and variance $(1 + a^2)C_X(0) - 2aC_X(s)$.

58. a) $E[Z(t)] = 0, \quad C_Z(t_1, t_2) = C_X(t_2 - t_1)\cos\omega(t_2 - t_1)$.

58. b) $Z(t)$ is Gaussian r.v. with mean zero and variance $C_X(0)$.

Chapter 7.

7.1 a) $S_X(f) = AT\left(\frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}\right)^2$.

7.1 b) $R_X(\tau) = AW\left(\frac{\sin\frac{\omega\tau}{2}}{\frac{\omega\tau}{2}}\right)^2$.

7.6 a) $R_{X,Y}(\tau) = R_{Y,X}(-\tau)$.

7.6 b) $S_{X,Y}(f) = S^*_{Y,X}(f)$.

7.21 a) $S_{X,Y}(f) = \frac{N_0/2}{1+j2\pi f}$.

7.21 b) $S_Y(f) = \frac{N_0/2}{1+4\pi^2 f^2}, \quad R_Y(\tau) = \frac{N_0}{4}e^{-|\tau|}$.

7.30 a) $S_{X,Y}(f) = \frac{(1+\beta e^{-j2\pi f})(1-\alpha^2)}{(1+\alpha^2-1\alpha\cos 2\pi f} \cdot \sigma^2$.

7.45 a) Use Eqn. 7.77

7.45 c) $h_0 = 0.767,, \quad h_1 = 0.072, \quad h_2 = 0.015.$

7.45 d) $E[e_t^2] = 0.773.$

7.46 a) $R_Z(k) = \begin{cases} (1+\alpha^2)\sigma^2, & k = 0, \\ \alpha\sigma^2, & k = \pm 1, \\ 0, & \text{otherwise.} \end{cases}$

7.46 b) $h_0 = \frac{\frac{(1+\alpha^2+\Gamma)^2(1+\alpha^2)-\alpha^2(1+\alpha^2)}{1+\alpha^2+\Gamma}-\alpha^2}{(1+\alpha^2+\Gamma)^2-2\alpha^2}.$

$h_1 = \frac{\alpha\Gamma}{(1+\alpha^2+\Gamma)^2-2\alpha^2}.$

$h_2 = \frac{\frac{-\alpha^2\Gamma}{1+\alpha^2+\Gamma}}{(1+\alpha^2+\Gamma)^2-2\alpha^2}.$

7.46 c) $E[e_t^2] = \sigma^2[(1+\alpha^2)(1-h_0) - \alpha h_1].$