Optional

September 9, 2004

Homework 1 in TMS115
Probability and Stochastic Processes, - Q.1, 2004/2005

• There are 10 total points in the homework. One needs 7.5 points for a bonus of 2 points out of 30 in the written examination.

• The dead-line for submission of the solution is 2004-09-24. Electronic submission of pdf or ps files is very welcome.

Problem 1. A company has three plants producing 50, 30, and 20 percent of the company product of electric relays, respectively. Suppose that a relay manufactured by these plants is defective with probability 0.02, 0.05, and 0.01, respectively.

(a) If a relay is selected at random from the output of the company, what is the probability that it is defective? (1)

(b) If a relay selected at random from the output of the company is found to be defective, what is the probability that it was not manufactured by plant 2? (1)

Problem 2. Suppose the bit error probability of a digital communication channel is 0.1. Assume the transmissions are independent.

(a) Find the probability that the third error occurs at the 10th bit. (1)

(b) Compute the average number of the transmitted bits until the third error occurs. (1)

Problem 3. Suppose X_1 and X_2 are jointly normal random variables with respective means μ_1 and μ_2, respective variances σ_1 and σ_2, and correlation ρ. What is $\mathbf{Pr}\{X_1 > X_2\}$? (1)

Problem 4. This academic year 135 applicants have been accepted into the International Master Program in Digital Communication Systems and Technology at Chalmers. However, only 45 students have been expected to come, based on previous experience. Use these information to estimate the probability that at least 40 students would attend the program. (1)

Problem 5. (See exercise problem 106 in Chapter 4 of Leon-Garcia's book.) A multiplexer combines N digital television signals into a common transmission line. Signal n generates X_n bits every 33 milliseconds, where X_n is a Gaussian random variable with mean m and variance σ^2. Suppose that the multiplexer accepts a maximum total of T bits from the combined sources every 33 ms, and that any bits in excess of T are discarded. Assume that the N signals are independent.

(a) Find the probability that bits are discarded in a given 33-ms period, if we let $T = m_a + t\sigma_a$, where m_a and σ_a are respectively the mean and the standard deviation of the total number of bits generated by the combined sources, and $t > 0$ is a fixed number. (1.5)

(b) Find the average number of bits discarded per 33-ms period. (1.5)

(c) Suppose that we require that t be adjusted with N so that the fraction of bits lost per source is kept constant. Find an equation for t. (1)