Optional

2004-09-24

Homework 2 in TMS115
Probability and Stochastic Processes, Q.1, 2004

- There are 10 total points in the homework. One needs 7.5 points for a bonus of 2 points out of 30 in the written examination.

- The dead-line for the submission of the solution is 2004-10-08. Electronic submissions of pfd or ps files are very welcome.

Problem 1. Let $X(t)$ be a Poisson process of parameter λ.

(a) Show that

$$P\{X(1) > 1,\, X(2) > 2,\, X(3) > 3\} = 1 - (1+\lambda)e^{-\lambda} - \frac{\lambda^2}{2}e^{-2\lambda} - \frac{2\lambda^3}{3}e^{-3\lambda}.$$

(2)

(b) For $t_1 < t_2 < \ldots < t_n$, compute the joint pmf $p_{X(t_1),\,X(t_2),\ldots X(t_n)}(x_1,\, x_2,\, \ldots x_n)$. (1)

(c) Compute the covariance function of the process $Y(t) = e^{-t}X(e^{2t})$. (2)

Problem 2. For the zero-mean stationary Gaussian process $X(t)$ with covariance function $C_X(\tau)$ compute:

(a) $P\{X(s) \geq X(t)\}$ in both cases when $C_X(t-s) \neq C_X(0)$ and $C_X(t-s) = C_X(0)$. (1)

(b) $E[X^2(t)X^2(t+\tau)]$. (2)

Problem 3. (See exercise problem 100 in Chapter 6 of Leon-Garsia's book.) A *Gauss-Markov random process* is a a Gaussian random process that is also a Markov process.

(a) Show that the autocovariance function of such a process must satisfy

$$C_X(t_3, t_1) = \frac{C_X(t_3, t_2)C_X(t_2, t_1)}{C_X(t_2, t_2)}.$$

where $t_1 \leq t_2 \leq t_3$. (1)

(b) It can be shown that if the autocovariance of a Gaussian random process satisfies the above equation, then the process is Gauss-Markov. Is the Wiener process Gauss-Markov? (1)