Written test for TMS115
"Probability and Stochastic Processes", 2004-10-22, Friday, 14–18, V.
Lecturer and on duty: Rossitza Dodunekova, tel. 772 3534. Time of visit 15:00 and 17:00.
Allowed material: Calculators approved by Chalmers, the handbook *Beta*.

There are 30 total points in the examination. One needs 14 points for grade 3 (to pass), 18 points for grade 4, and 24 points for grade 5.

Problem 1. The random vector $\mathbf{X_{2n}} = (X_1, X_2, \ldots, X_{2n})$ has independent coordinates which are Bernoulli random variables of parameter p, $0 < p < 1$. Let the random variable Y_{2n} be the number of ones in $\mathbf{X_{2n}}$. Compute

$$\lim P\{Y_{2n} \le n\}, \quad \text{when} \quad n \to \infty.$$

(3)

Problem 2. Suppose Z_1 and Z_2 are jointly Gaussian random variables with joint pdf

$$f_{Z_1, Z_2}(z_1, z_2) = \frac{1}{\sqrt{2}\pi} e^{-(z_1^2 - \sqrt{2} z_1 z_2 + z_2^2)}.$$

(a) Compute $Cov\,(Z_1 - Z_2/\sqrt{2},\ Z_2)$. (2)

(b) Compute $E[\,Z_1^2 Z_2]$. (2)

Problem 3. Let X be the number of active speakers in a group of M independent speakers, each one of which is active with probability p. Suppose that a voice transmission system can transmit up to $N < M$ voice signals at a time, and that when X exceeds N, $X - N$ randomly selected signals are discarded.

(a) Give a formula for computing the expected value of the discarded voices. (2)

(b) Estimate the probability that voices are not discarded if $M = 45,\ p = 1/3,\ N = 16$. (2)

Problem 4. Messages arrive at a computer from two telephone lines according to two independent Poisson processes of rate λ and μ, respectively.

(a) Compute the probability that a message arrives first on line 1. (2)

(b) Compute the pdf of the waiting time for the first message to come. (2)

(c) Assume the total number of messages in $[0, t]$ is three. Compute the conditional probability that at least one message has arrived in the first halve of the interval and at least one in the second. (2)

Problem 5. $X(t)$ and $Y(t)$ are jointly wide-sense stationary random processes and $Z(t)$ is defined by

$$Z(t) = bX(t) - Y(t - b),$$

where b is a non-zero constant. Determine whether or not $Z(t)$ is wide-sense stationary. (2)

Problem 6. Let $Y_n = X_n + \beta X_{n-1}$, where X_n is a zero-mean wide-sense stationary random process with autocorrelation $R_X(k) = \sigma^2 \alpha^{|k|}, \quad |\alpha| < 1$.

(a) Find $S_{Y,X}(f)$ and $R_{Y,X}(k)$. (2)

(b) Find the values of β for which is Y_n a white-noise process. (3)

Problem 7. Suppose $\{Y_n\}$ is defined as

$$Y_n = \frac{1}{2} Y_{n-1} + W_n,$$

where W_n is the white-noise process of average power σ_W^2.

(a) Compute the autocorrelation function of Y_n. (3)

(b) Let $\sigma_W^2 = 3$. Compute the best linear estimation of Y_n from Y_{n-2} and Y_{n-3} and the mean-square error of estimation. (3)