Optional

Homework 1 in TMS115
Probability and Stochastic Processes, 2005/2006

• There are 10 total points in the homework. One needs 7.5 points for a bonus of 2 points out of 30 in the written examination.

• The dead-line for submission of the solution is 2005-09-21. Electronic submission of pdf or ps files is very welcome.

Problem 1. A multiple-choice exam gives 5 choices per question. On 75% of the questions, you think you know the answer; on the other 25% of the questions, you just guess the answer. Unfortunately when you *think* you know the answer , you are wright only 80% of the time.

(a) Find the probability of getting an arbitrary question right. (1)

(b) If you do get a question right, what is the probability that it was a lucky guess? (1)

Problem 2. A disadvantage of a broadcast subnet is the capacity wasted due to multiple hosts attempting to access the channel at the same time. Suppose a time period is divided into a certain number of discrete slots. During each time slot, the probability is p that a host will want to use the channel. If two or more hosts want to use a time slot, then a collision occurs, and the slot is wasted.

(a) If there are n hosts, what fraction of the slots is wasted due to collisions? (1)

(b) Assume that $p = 5/n + 1/n^2$. Find a numerical approximation of the result in a) for large values of n. Explain. (1)

Problem 3. The number X of electrons counted by a receiver in an optical communication system is a Poisson random variable with rate λ_1 when a signal is present and with rate $\lambda_0 < \lambda_1$ when a signal is absent. Suppose that a signal is present with probability p.

(a) Find $P\{signal\ present\,|X = k\}$ and $P\{signal\ absent\,|X = k\}$. (1)

(b) The receiver uses the following decision rule:

If $P\{signal\ present\,|X = k\} > P\{signal\ absent\,|X = k\}$, decide signal present;

otherwise decide signal absent.

Show that this decision rule leads to the following threshold rule:

If $X > T$, decide signal present; otherwise, decide signal absent. (1)

(c) What s the probability of error for the above decision rule? (1)

Problem 4. The lifetime X of a device is an exponential random variable with mean value $1/R$. Suppose that due to the irregularities in the production process, the parameter R is random and has Gamma distribution.

(a) Find the joint pdf of X and R. (1)

(b) Find the pdf of X. (1)

(c) Find the mean and variance of X. (1)