

Optional

Homework 2 in TMS115 Probability and Stochastic Processes, 2005/2006

- There are 10 total points in the homework. One needs 7.5 points for a bonus of 2 points out of 30 in the written examination.
 - The dead-line for submission of the solution is 2005-10-14. Electronic submission of pdf or ps files is very welcome.
 - Please put your **surname first**.
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Problem 1. The random variables X_1 , X_2 , and X_3 are jointly Gaussian. Let

$$Y_1 = X_1 + X_2, \quad Y_2 = X_2 + X_3, \quad Y_3 = X_1 + X_3.$$

Show that Y_1 , Y_2 , and Y_3 are jointly Gaussian. (2)

Problem 2. Consider a Poisson process $X(t)$ with rate λ , and suppose that each time an event occurs it is classified as either a type 1 or a type 2 event. Suppose further that the event is classified as a type 1 event with probability p and a type 2 event with probability $1 - p$. Let $X_1(t)$ and $X_2(t)$ denote the number of type 1 and type 2 events, respectively, occurring in $[0, t]$.

- Show that the processes $\{X_1(t), t \geq 0\}$ and $\{X_2(t), t \geq 0\}$ are both Poisson processes with rates λp and $\lambda(1 - p)$, respectively. (2)
- Show that $\{X_1(t), t \geq 0\}$ and $\{X_2(t), t \geq 0\}$ are independent. (2)

Problem 3. Consider the random process

$$Y(t) = (-1)^{X(t)}$$

where $X(t)$ is a Poisson process with rate λ .

- Find the mean-value time function and the autocorrelation function of $Y(t)$. Is the process WSS? (2)
- Consider the process $Z(t) = AY(t)$, where A is a random variable, independent of $Y(t)$ and taking on values ± 1 with equal probabilities. Is $Z(t)$ WSS? (1)
- Find the power spectral density of $Z(t)$. (1)