Written test for the examination
"Probability and Stochastic Processes", 2005-01-14, 14:00 - 18:00, house V.
Lecturer and on duty: Rossitza Dodunekova, 772 3534.
Allowed material: Calculators with empty memories, the handbook *Beta*.

There are 30 total points in the examination. One needs 14 points for grade 3 (to pass), 18 points for grade 4, and 24 points for grade 5.

Problem 1. A binary communication channel with $\varepsilon_i \le 1/2$ is shown in the figure below. Let the input probability of 1 be p, where $0 < p < 1$.

(a) Assume first that $p = 1/2$ and $\varepsilon_i < 1/2$. Given that the output is 1, which input is more probable? (1.5)

(b) Let p be any number in (0, 1) and the channel be symmetric, i.e., $\varepsilon_1 = \varepsilon_2 = \varepsilon \le 1/2$. Find the value of ε for which input and output are independent. Is such a channel appropriate for information transmission? (1.5)

Input			*Output*
0		$1 - \varepsilon_1$	0
	ε_1		
	ε_2		
1		$1 - \varepsilon_2$	1

Problem 2. Messages arrive in a multiplexer according to a Poisson process of rate 10 messages per second. Use the CLT to estimate the probability that more then 600 messages arrive in one minute. (3)

Problem 3. X and Y are independent random variables uniformly distributed in (0,1).

(a) Compute $E[(X - Y)^2]$ (2)

(b) Let $Z = X + Y$. Find the pdf of Z. (2)

(c) Compute $E[Z|Z > 1]$. (2)

Problem 4. $N(t)$ is the Poisson process with parameter λ.

(a) Find the joint pmf of $N(1)$ and $N(4)$. (2)

(b) Compute $P\{N(1) > 1,\ \ N(4) < 4\}$. (2)

(c) Compute the covariance function of the process $X(t) = e^{-t}N(e^{2t})$. (3)

Problem 5. X is a WSS random process with autocorrelation function

$$R_X(\tau) = 16e^{-5|\tau|} \cos 2\pi\tau + 8 \cos 10\pi\tau.$$

Compute the power spectral density of X. What are the average power at zero frequency and the average power of X? (4)

Problem 6. A linear system has an impulse response of the form

$$h(t) = \begin{cases} 5\delta(t) + 3, & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

The input to this system is a random sample function of the form

$$X(t) = 2cos(2\pi t + \theta), \quad -\infty < t < \infty,$$

where θ is a random variable uniformly distributed in $(0, 2\pi)$.

(a) Find an expression for the output sample function $Y(t)$. (2)

(b) Find m_Y and $Var(Y(t))$. (2)

Problem 7. X is WSS with autocorrelation function

$$R_X(k) = 9(1/3)^{|k|}, \quad k = 0, \pm 1, \pm 2, ...$$

(a) Find the optimum linear filter for estimating X_n from the observations X_{n-2} and X_{n+1}. (2)

(b) Compute the mean-square estimation error in (a). (1)