Written test for examination in TMS115
TMS115 Probability and Stochastic Processes, 2006-01-13, Friday, 14:00 - 18:00, V.
Lecturer and on duty: Rossitza Dodunekova, tel. 772 3534.
Time of visit 15:00 and 17:00.
Allowed material: Calculators approved by Chalmers, the handbook *Beta*.

There are 30 total points in the examination. One needs 12 points for grade 3 (to pass), 18 points for grade 4, and 24 points for grade 5.

Problem 1. Suppose a terminal produces messages every millisecond according to a sequence of independent Bernoulli trials with success probability p. Let N be the number of times the computer polls the terminal until the terminal has a message ready for transmission.

(a) Derive the pmf of N and evaluate its expectation. (2)

(b) Assume the computer polls 10 times in vain. How many polls on the average are needed from now on to get a message? (2)

Problem 2. The lifetime X of an electronic component has an exponential distribution such that $P\{X \leq 1000\} = 0.75$.

(a) What is the expected lifetime of the component? (2)

(b) 10 such components are put into function at the same time. Find the probability that at least 5 or more will survive at least 1000 time units. (2)

Problem 3. Suppose Z_1 and Z_2 are independent standard normal random variables.

(a) What is the joint distribution of $X_1 = Z_1$ and $X_2 = 3/5Z_1 + 4/5Z_2$? Explain. (2)

(b) Compute $f_{X_2|X_1}(x_2|x_1)$, the conditional pdf of X_2, given $X_1 = x_1$. (2)

Problem 4. The number of applicants accepted to the master's program in Digital Communication Systems is 48. It is known from the previous experience that an accepted student would in fact participate with probability 2/3, thus the expected number of students in the program is 32. Use the Central Limit Theorem to estimate the probability that at most 22 student will attend the program. (2)

Problem 5. Let $\{X(t)\}$ and $\{Y(t)\}$ be independent, wide-sense stationary random processes with zero means and the same correlation function $R(k)$. Let $Z(t)$ be defined by

$$Z(t) = aX(t) + bY(t-d).$$

where a and b are nonzero constants.

(a) Compute $S_Z(f)$ and $S_{Z,X}(f)$. (2)

(b) Determine the pdf of $Z(t)$ if $X(t)$ and $Y(t)$ are also jointly Gaussian random processes. (2)

Problem 6. A moving average process X_n is produced as follows:

$$X_n = W_n + \alpha_1 W_{n-1} + \alpha_2 W_{n-2} + \alpha_3 W_{n-3}$$

where W_n is a zero-mean white noise process of average power 1.

(a) Compute $R_X(k)$. (2)

(b) Find $S_X(f)$ and the transfer function of the filter that produces the moving average process. (2)

Problem 7. $X(t)$ is a zero mean WSS process with autocorrelation function

$$R_X(k) = 4(1/2)^{|k|}, \quad k = 0, \pm 1, \pm 2, \ldots.$$

(a) Derive the equiations which determine a and b in the best linear predictor of X_n, $\hat{X}_n = aX_{n-1} + bX_{n-3}$. Explain. (2)

(b) Derive a formula for computing the the mean-square error in (a). Explain. (2)

Problem 8. $X(t)$ is a zero-mean WSS process with autocorrelation function $R_X(\tau)$. It is passed through a linear time-invariant system with unit impuls response $h(t)$, and the output of the system is $Y(t)$. Let $Z(t) = X(t) - Y(t)$.

(a) Find $S_Z(f)$ in terms of $S_X(f)$. (2)

(b) Find $E[Z^2(t)]$. (2)