Statistical inference for Geometrical BM (cf. Exercise 10.9) dX = mu X dt + X dB ; X (t) = Exp {(mu - 1 / 2) t + B (t)}

```
In[1]:= << Statistics`DescriptiveStatistics`; << Statistics`ContinuousDistributions`;
```

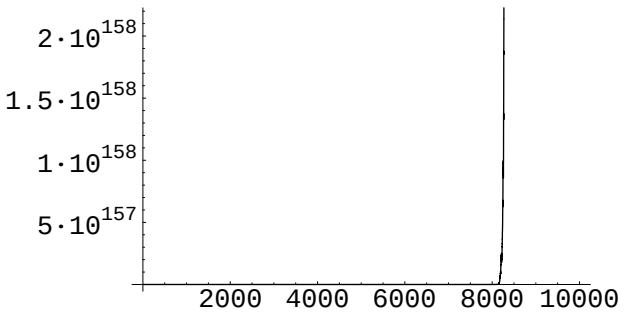
The transition density function for the OU process pBM[mu, x, y, t] = P {BM (t) + (mu - 1 / 2) t ∈ dy | B (0) = x} dy :

```
In[2]:= Clear[pBM]; pBM[mu_, x_, y_, t_] :=
         PDF[NormalDistribution[x + (mu - 1 / 2) * t, Sqrt[t]], y];
```

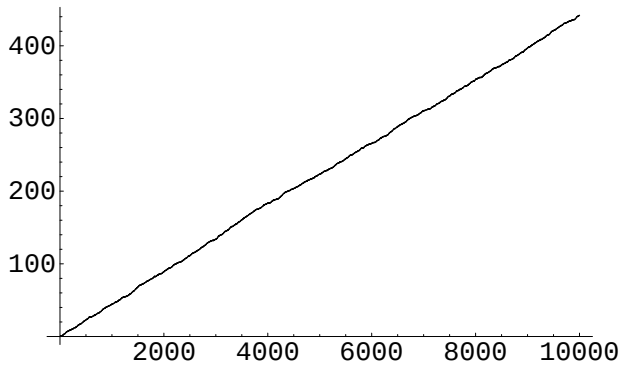
Simulation of Geometrical BM is straightforward by the Euler method :

```
In[21]:= mu0 = 5; StepsPerTimeUnit = 100; T = 100;
         GBMData = {1}; For[i = 2, i ≤ StepsPerTimeUnit * T, i++,
          AppendTo[GBMData, GBMData[[i - 1]] + mu0 * GBMData[[i - 1]] / StepsPerTimeUnit +
            GBMData[[i - 1]] * Random[NormalDistribution[0, 1 / Sqrt[StepsPerTimeUnit]]]]];
```

```
In[22]:= ListPlot[GBMData, PlotJoined → True];
```




```
In[23]:= ListPlot[Log[GBMData], PlotJoined → True];
```



Method 1 : Estimation of mu by Likelihood Ratio according to Equation (10.42)

```
In[24]:= mu1 = Sum[(1 / GBMData[[i - 1]]) * (GBMData[[i]] - GBMData[[i - 1]]),
            {i, 2, StepsPerTimeUnit * T}] / T
Out[24]= 4.99675
```

Method 2 : Estimation of mu by Maximum Likelihood method based on pBM[mu, x, y, t]

In[25]:=
```
SampleDistance = 50; NData = StepsPerTimeUnit * T / SampleDistance;
NewGBMData = Table[GBMData[[i * SampleDistance]], {i, 1, NData}];
mu2 = mu /. NMaximize[{Sum[Log[pBM[mu, Log[NewGBMData[[i - 1]]],
        Log[NewGBMData[[i]]], T / NData]], {i, 2, NData}], mu > 0}, {mu}][[2]]
```

Out[25]=
```
4.92577
```