**Exercise session 2, Stochastic Calculus Part I.**

**1** Let $X$ be a standard normal r.v.

1. Calculate $\mathrm{E}[\,X \mid X = x]$.
2. Calculate $\mathrm{E}[\,X \mid X > 0]$.
3. Calculate $\mathrm{E}[\,|X| \mid X < 0]$.

**2** Let $\{X_t\}_{t\geq 0}$ be a stochastic process with independent Gaussian increments, i.e., for $s < t$, $X_t - X_s$ is normally distributed and independent of $X_u$ for $u \leq s$. Show that $X_t$ is a Gaussian process.

A stochastic process $X_t$ has independent increments if $X_{t_n} - X_{t_{n-1}}, \ldots, X_{t_2} - X_{t_1}$ are independent, for all $t_1 < t_2 < \ldots < t_n$. $X_t$ has stationary increments if $X_{t+s} - X_s$ does not depend on $s$. A stochastic process with independent stationary increments is called a *Lévy* process.

**3** Let $X_t$ and $Y_t$ be independent Lévy processes.

a) Is $X_t + Y_t$ a Lévy process?

b) Can a Lévy process be a Martingale?

c) Let the increments $X_t - X_s$ be Normal(0,$t - s$). Calculate $\mathrm{Cov}(X_t, X_s)$.

d) Simulate and plot a realization of $X_t$ on $[0, 1]$.

**4** Let $X > 0$ be a random variable and $\mathrm{E}[X] < \infty$. Show that $\mathrm{E}[\max(0, X - n)] \to 0$ as $n \to \infty$.

**5** Properties (2.16)-(2.27) in Klebaner of conditional expectation.