Exercise session 3, Stochastic Calculus Part I.

1 Let $X > 0$. Show that $\mathrm{E}[X] \leq \sum_{n=o}^{\infty} P(X > n)$.

2 Let B_t be a Brownian motion. Show that $X_t = cB(t/c^2)$ is a Brownian motion.

3 Let B_t be a Brownian motion. Show that $e^{-\alpha t} B(e^{2\alpha t})$ is a Gaussian process. Find its mean and covariance functions.

4 Let B_t be a Brownian motion. Show that the process $e^{-t/2} \cosh(B_t)$ is a martingale w.r.t. the filtration $\mathfrak{F}_t = \sigma(B_s, 0 \leq s \leq t)$.

5 Calculate $P(tB_1 + (1-t)B_2 \leq 1)$.