

Stochastic Calculus II, 2007

Hand in 3

1. Let $B(t)$ be a Brownian motion.

Find a process $\tau(t)$ such that $B(\tau(t)) = t$.

Motivate why (by which property of Brownian motion) $B(\tau(t))$ really equals t (almost surely). (2p.)

2. Invent your own exercise 7.13. That is let $M(t) = \int_0^t f(s) dB(s)$ for a suitable function f of your own choice (but do not choose $f(s)$ constant, $f(s) = s$ or $f(s) = e^s$) and then find a function g such that $M(g(t))$ is a Brownian motion.

Does f have to be injective?

Does g have to be non-decreasing?

Does g have to be increasing? (4p.)

3. Let $M(t) = \int_0^t \frac{1}{1+s^2} dB(s)$. Is it possible to find a function g such that $M(g(t))$ is a Brownian motion? (As usual you should motivate your answer!). (2p.)