Lab 2 - Linear Mixed Models of Longitudinal Data

1 Introduction

In this second computer assignment we consider a dataset with measurements of blood pressure from 15 patients. The patients have moderate essential hypertension (high blood pressure) and their systolic and diastolic blood pressure are measured before and 2 hours after taking the drug captopril. The objective of the lab is to investigate the effect of treatment on both responses.

Note that there is a double structure in the data,

- a **bivariate** structure since both systolic and diastolic blood pressure is measured,
- **repeated measures**, before and after treatment.

This data is analyzed in Chapter 24.1 in Verbeke and Molenberghs. It is recommended that you read this text before you start.

2 Modeling

You should compare the final model (24.2) in Verbeke and Molenberghs with three different models using covariance structures for multivariate repeated measures.

1. **4x4 unstructured covariance matrix:**

$$Y_{ij} = \begin{cases} \beta_1 + \varepsilon_{i,1} & \text{diastolic, before} \\ \beta_2 + \varepsilon_{i,2} & \text{systolic, before} \\ \beta_3 + \varepsilon_{i,3} & \text{diastolic, after} \\ \beta_4 + \varepsilon_{i,4} & \text{systolic, after,} \end{cases}$$

   where ε has an unstructured covariance structure,

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} & \sigma_{34} \\ \sigma_{14} & \sigma_{24} & \sigma_{34} & \sigma_{14} \end{pmatrix}$$

2. **Unstructured by unstructured**

$$Y_{ij} = \begin{cases} \beta_1 + \varepsilon_{i,0,0} & \text{diastolic, before} \\ \beta_2 + \varepsilon_{i,1,0} & \text{systolic, before} \\ \beta_3 + \varepsilon_{i,0,1} & \text{diastolic, after} \\ \beta_4 + \varepsilon_{i,1,1} & \text{systolic, after,} \end{cases} \tag{1}$$

and the covariance structure is the Kronecker product of two unstructured covariance matrices, one for time and one for the different measurements,

$$\text{Cov}(\varepsilon_{i,j_1,j_2}, \varepsilon_{i,k_1,k_2}) = \sigma_{1,j_1,k_1}\sigma_{2,j_2,k_2}$$

where the j_1, k_1 index indicates the type of measurement (diastolic or systolic), and the j_2, k_2 corresponds to the timepoints.

3. **Unstructured by compound symmetry** Here you should use the same model as in (1) but with the following covariance structure,

$$\text{Cov}(\varepsilon_{i,j_1,j_2}, \varepsilon_{i,k_1,k_2}) = \sigma_{j_1,k_1}(1 - \sigma^2 1_{(j_2 \neq k_2)}),$$

where $0 \leq \sigma^2 \leq 1$.

Questions:

1. Compare the four models, which one is the best? Discuss.
2. Discuss the pros and cons of each of the models.
3. Would it make sense to make a model with serial correlation? Why is the variable `time` included in the `class` statement?
4. Estimate and interpret the treatment effects for both diastolic and systolic blood pressure. Do the treatment improve the blood pressure of the patients?

3 Summary

Please write a short report where you discuss the above questions describe your findings.