Petter Mostad
Mathematical Statistics
Chalmers

MSA830 Statistical analysis and experimental design

Exam 22 October 2010
Examiner: Petter Mostad, phone 0707163235,
visits the exam at 9.30 and at 11.00.
Allowed to use during the exam: Pocket calculator, books, copies, and notes
Number of points on the exam: 30. To pass the exam, at least 12 points are needed

1. Ralph would like to test the fitness effect of his specially designed training program. Each of 5 test persons takes a special fitness test before and after the training program, and the scores are given in the table below:

	Person 1	Person 2	Person 3	Person 4	Person 5
Before program	104	93	121	88	73
After program	114	107	123	111	92

   (a) Find a 95% credibility interval for the increase in the fitness score the training program seems to produce. (Make the assumptions you need in order to make your calculations). (3 points)

   (b) What assumption did you need to make the calculations? Assume Ralph goes on to test a total of 25 persons. Mention at least one thing Ralph could do to see if the assumption is OK for his data. (1 point)

   (c) If Ralph does not want to make the assumption mentioned above, describe briefly a test he could perform instead, where he would be testing whether it could be random which one of the two scores for each person was before or after the training program. (You do not have to do any computations) (1 point)

   (d) Ralph has selected his test persons from persons who have newly joined a fitness club. Discuss wheter Ralph has really shown that his training program increases a persons fitness. Give an argument against that this has been shown. Can you propose an alternative experimental plan with which Ralph could better measure the effectiveness of his program? (1 point)

2. Roza runs a website which she would like to make as attractive as possible for visiting customers. The main page has three different areas the visitor can click on to continue the visit: A, B, and C. The visitor may also leave the site without clicking on any of the areas. Out of 1000 unique visitors, the table below shows what each visitor chose:

A	B	C	None
327	34	128	511

   Roza assumes that each visitor acts independently of the others. She decides to use the empirical distribution to get probabilities predicting what new visitors will do.

   (a) What is the probability for the next visitor to click on B? (1 point)

(b) What is the probability that, among the 10 next visitors, exactly 2 will click on C? (2 points)

(c) What is the approximate probability that, among the next 100 visitors, 50 or more will click on A? (2 points)

(d) Roza later finds out that, out of visitors who click on A, 10% go on to buy something. Out of visitors who click on B, 5% go on to buy. Out of visitors who click on C, 2% go on to buy. (Clearly visitors who just leave the site do not buy anything). Given that a visitor has ended up buying something, what is the probability that the visitor first clicked on A? (2 points)

3. Ulla works in a materials lab, and is studying how the durability of a new material depends on the amount of an additive used, and the temperature during manufacture. She does an experiment where she tests out three levels, A, B, and C, for the amount of the additive, and three temperatures, X, Y, and Z. For each combination of these factors she does two replicated experiments, and measures the durability of the result, obtaining in this way 18 durability measurements. Her results are given in the table below (higher values means higher durability). The averages of different groups of six numbers have been computed, for your convenience. The grand average appears at the bottom right corner. The variance of all the data is 18.81.

	A	B	C	Average
X	40 32	43 40	28 34	36.17
Y	34 32	36 38	34 34	34.67
Z	38 35	44 40	40 42	39.83
Average	35.17	40.17	35.33	36.89

(a) Make a complete ANOVA table for Ulla's experiment. In this analysis, do not include interaction between the factors. For the p-values: Find an interval containing each p-value. (4 points)

(b) Explain the results of the analysis in terms and words that can be understood by somebody who knows nothing about ANOVA tables or statistics. Include also recommendations in terms of temperatures and amount of additive. (1 point)

(c) Start to fill out a new ANOVA table for the case where interactions between factors is included. You only have to fill out the "degrees of freedom" column and then describe how you would do the calculations to fill out the sum of squares column. (2 points)

(d) Ulla is worried that the model she has used for her analysis (either with or without interaction) is not appropriate for her data. Mention at least one test or plot that she could make to investigate this question. (1 point)

4. Karim would like to study how various factors influence the taste of his muffins, as judged by a tasting panel. He would like to study 5 factors, A, B, C, D, E, each with two possible levels. In other words, for each batch of muffins he makes, he will be able to independently set each of the factors at one of its levels (written + and -). He would like to make 8 batches

of muffins. After all experiments have been performed, he would like to do an analysis where he can independently estimate the effect of each factor, and also the interaction effect between A and B[1].

(a) Set up an experimental plan for Karim, satisfying his requirements. (2 points)

(b) Karim would like to compute a credibility interval for the effect of factor B. Does it matter for the computation of this credibility interval whether he includes the interaction effects in his analysis or not? In which way does it matter? (1 point)

(c) Write down the "name" of your experimental plan, in some format like 2^* or 2^*_*. What is the resolution of your design? (1 point)

(d) To the extent that Karim's data indicate how the factors influence the taste, he would like his results to to be as reproducible as possible, if others try to vary the same factors. Give at least three advice to Karim about how he should perform his experiments, in order for his results to be as reproducible as possible. (1 point)

5. Esther has made repeated independent measurements of the concentrations of pollutant X at four different locations: A, B, C, and D. Her results are given in the table below, together with summary statistics for each group:

	Observations	N. of obs.	Mean	Variance
Location A	17.3, 13.5, 11.8, 13.0, 13.9, 14.6, 13.4	7	13.93	2.94
Location B	21.7, 14.9, 21.4, 18.3, 16.4	5	18.54	9.01
Location C	19.2, 17.6, 18.0, 17.5, 16.4, 17.9, 17.9, 16.2, 17.6	9	17.59	0.79
Location D	20.4, 22.4, 21.6, 15.0, 23.0, 21.8, 19.7, 21.4, 22.8, 19.2	10	20.73	5.66

(a) Assuming that all four groups of measurements come from normal distributions with the same precision (but with different expectations), find a 95% credibility interval for the difference between the expectations for measurements at location A and C. (1 point)

(b) Under the assumptions above, find a 95% credibility interval for the common precision in the four groups of observations. (1 point)

(c) Making only the assumption that measurements at locations A and C come from normal distributions with the same precision (but with different expectations), find again a 95% credibility interval for the difference between expectations for the measurements at locations A and C. (1 point)

(d) Making now only the assumption that measurements at locations A and C come from normal distributions (which may have different precisions), find again a 95% credibility interval for the difference between expectations for the measurements at locations A and C. (1 point)

[1] In the original exam, Karim was also required to be able to independently estimate the interaction effect between D and E. However, no experimental plan satisfying all these requirements exist