Petter Mostad
Matematisk Statistik
Chalmers

Suggested solution for exam in
MSA830: Statistical Analysis and Experimental Design
10 January 2012

1. (a) The mean and variance of the 5 ages is 32.6 and 132.8, respectively. The expected (average) age μ of the normal distribution these ages come from then has the t-distribution

$$\mu \sim \mathrm{t}(32.6, 4, \log(\sqrt{132.8/5})) = \mathrm{t}(32.6, 4, \log(5.1536))$$

A 95% credibility interval becomes

$$\begin{aligned} & [32.6 - t_{4,0.025}5.1536, 32.6 + t_{4,0.025}5.1536] \\ = \ & [32.6 - 2.7764 \cdot 5.1536, 32.6 + 2.7764 \cdot 5.1536] = [18.3, 46.9] \end{aligned}$$

(b) The logged scale λ has distribution

$$\lambda \sim \mathrm{ExpGamma}(\frac{4}{2}, \frac{4}{2}132.8, -2) = \mathrm{ExpGamma}(2, 265.6, -2)$$

so the standard deviation e^λ has 95% credibility interval

$$\left[\sqrt{\frac{531.2}{\chi^2_{0.025,4}}}, \sqrt{\frac{531.2}{\chi^2_{0.975,4}}}\right] = \left[\sqrt{\frac{531.2}{11.143}}, \sqrt{\frac{531.2}{0.484}}\right] = [6.9, 33.1]$$

(c) We compute that

$$(19 - 37.9)^2 + (37 - 37.9)^2 + (33 - 37.9)^2 + (49 - 37.9)^2 + (25 - 37.9)^2 = 671.65$$

so the logged scale λ now has the distribution

$$\lambda \sim \mathrm{ExpGamma}(\frac{5}{2}, \frac{1}{2}671.65, -2) = \mathrm{ExpGamma}(2.5, 335.825, -2)$$

so the standard deviation e^λ has 95% credibility interval

$$\left[\sqrt{\frac{671.65}{\chi^2_{0.025,5}}}, \sqrt{\frac{671.65}{\chi^2_{0.975,5}}}\right] = \left[\sqrt{\frac{671.65}{12.833}}, \sqrt{\frac{671.65}{0.831}}\right] = [7.2, 28.4]$$

2. In this case, we have two sets of observations, both from normal distributions with known scales: The variance is $0.37^2 = 0.1369$. We then get that the difference in luminosity has distribution

$$\mathrm{Normal}\left(61.4 - 56.2, \log\left(\sqrt{0.1369/7 + 0.1369/4}\right)\right) = \mathrm{Normal}\left(5.2, \log(0.2319)\right)$$

A 90% credibility interval becomes

$$[5.2 - 1.64 \cdot 0.2319, 5.2 + 1.64 \cdot 0.2319] = [4.8, 5.6]$$

3. (a) He should look at data from each of the species separately. For each he can check normality visually, by for example looking at histograms, or he can make hypothesis tests, such as for example the Shapiro test.

   (b) The test statistic to use is

$$F = 0.96/0.87 = 1.103$$

   which should be compared with an F distribution with 18 and 15 degrees of freedom. From the appropriate table, we find that the probability for such a distribution to be above 1.103 is above 0.25, so the p-value is above 0.5. Thus Joachim should *not* reject the null hypothesis that the standard deviations of the two normal distributions are equal.

   (c) Joachim assumes that the standard deviations in the two normal distributions are equal. He thus computes the pooled variance

$$s_p^2 = \frac{14 \cdot 0.87 + 17 \cdot 0.96}{14 + 17} = 0.9194$$

   and the test statistic

$$t = \frac{2.95 - 2.31}{\sqrt{0.9194\,(1/15 + 1/18)}} = 1.9092$$

   and compares it to the standard t-distribution with $15+18-2 = 31$ degrees of freedom. The probability from the table is above 0.025, so the p-value, which is twice this, is above 0.05. From this, one can *not* reject the null hypothesis that the expectations of the two normal distributions are the same. The result is that based on the data Joachim has, he can not conclude that the normal distributions of the sizes of animals of the two species are different.

4. (a) With Sally's assumptions, the number of accidents next year is Poisson distributed with rate $137/30 = 4.567$. The probability that there will be no accidents next year is thus

$$\frac{1}{0!}e^{-4.567}4.567^0 = e^{-4.567} = 0.01$$

   (b) The probability that there will be 5 accidents is

$$\frac{1}{5!}e^{-4.567}4.567^5 = 0.17$$

   (c) We make a normal approximation: The Poisson distribution above has expectation 4.567 and variance 4.567, thus we compare with a normal distribution with this expectation and variance. We then compare

$$\frac{9.5 - 4.567}{\sqrt{4.567}} = 2.308$$

   with the standard normal distribution. We find from the table the approximate probability 0.01.

5. The probability can be computed as

$$\begin{aligned}
\pi(\text{passing}) &= \pi(\text{passing, subject A}) + \pi(\text{passing, subject B}) + \pi(\text{passing, other}) \\
&= \pi(\text{passing} \mid \text{subject A})\pi(\text{subject A}) + \pi(\text{passing} \mid \text{subject B})\pi(\text{subject B}) + \\
&\quad \pi(\text{passing} \mid \text{other})\pi(\text{other}) \\
&= 0.9 \cdot 0.4 + 0.7 \cdot 0.3 + 0.1 \cdot (1 - 0.4 - 0.3) \\
&= 0.6
\end{aligned}$$

So Günther has a 60% chance of passing the exam.

6. (a) We get the following sums of squares:

$$\begin{aligned}
SS_{\text{Temp}} &= 18(86.94 - 87.83)^2 + 18(89.61 - 87.83)^2 + 18(86.94 - 87.83)^2 = 85.5468 \\
SS_{\text{Press}} &= 18(87 - 87.83)^2 + 18(87.5 - 87.83)^2 + 18(89 - 87.83)^2 = 39.0006 \\
SS_{\text{Machine}} &= 18(87.5 - 87.83)^2 + 18(88.78 - 87.83)^2 + 18(87.22 - 87.83)^2 = 24.903 \\
SS_{\text{Total}} &= 53 \cdot 9.1981 = 487.4993
\end{aligned}$$

From this, we get the ANOVA table

	SS	D.f.	M.sq.	F	p
Temperature	85.5468	2	42.7734	5.94692	$p < 0.01$
Pressure	39.0006	2	19.5003	2.71119	$0.05 < p < 0.1$
Machine	24.903	2	12.4515	1.73117	$0.1 < p < 0.25$
Residuals	338.0489	47	7.19253		
Total	487.4993	53			

The conclusions are that the temperature has a significant influence on the yield; according to the averages, temperature B gives the highest yield. However, neither the Pressure or the machine can be seen to have a significant influence.

(b) We get that

$$SS_{\text{Pres:Temp}} = 6\Big((86.11 - 87.83)^2 + (88.78 - 87.83)^2 + (86.11 - 87.83)^2 + (86.61 - 87.83)^2 + (89.28 -$$

which leads to the ANOVA table

	SS	D.f.	M.sq.	F	p
Temperature	85.5468	2	42.7734		
Pressure	39.0006	2	19.5003		
Interaction	124.5474	4	31.1368	6.271	$p < 0.01$
Machine	24.903	2	12.4515		
Residuals	213.5015	43	4.96515		
Total	487.4993	53			

As the p-value is smaller than 0.05, it is reasonable to include interaction in the model. In other words, one should consider each combination of pressure and temperature separately.

(c) The assumptions are that the observed values are given as linear combinations of the effect parameters for each of the factors plus error terms that independent and normally distributed. To check such assumptions one can study the residuals, which are the differences between the observed values and the fitted values of the model. These residuals should be approximately independent and normally distributed. To check this, one can make plots of the residuals agains the values of the factors, and against time. In the plots, the residuals should show independence and appear roughly normally distributed.

7. Such an experimental plan is shown below:

A	B	C	D	E	F	G
-	-	-	+	+	+	-
-	-	+	+	-	-	+
-	+	-	-	+	-	+
-	+	+	-	-	+	-
+	-	-	-	-	+	+
+	-	+	-	+	-	-
+	+	-	+	-	-	-
+	+	+	+	+	+	+

8. Some problems are

- The measurement of the result, i.e., the number of days before the bread tastes bad, is made in a way that does not propote consistency. It is done by several different bakeries, which may have different opinions of where the limit for bad taste is. Instead, such tasting should be done centrally, possibly by one single person.
- The changes of the amount of ingredient X is confounded with possible differences there might be between the bakeries. There are likely to be some such differences, and the effects of these differences cannot be separated from the effects of ingredient X. Instead one should randomize, or block, which experiments use which amounts of X.
- the changes in the cool-down time are confounded with the time factor: It might be that the measurement of bad taste would change over time, and the effects of such changes would with Harri's setup be counfounded with the effects of changes of the cool-down time. One should use randomization or blocking instead.
- The number of experimental runs is far too small: With no replications, it would be very difficult to get any significant results from this experiment.

9. (a) As there are three parameters, there must be three numbers giving the least squares estimates. As the fitted values are linear combinations of the values of x_1 and x_2 and the least squares estimates, the least squares estimates cannot all be negative numbers, as that would mean that the fitted values would be negative: They cannot be as the observations are positive numbers; they are weights of something. Thus the correct answer must be (ii).

   (b) As we have 4 experimental runs, there must be 4 fitted values. The values in (iii) cannot be the answer: If three of the fitted values are identical, then the fourth fitted value must also be equal to the others, as we have no interaction Thus the correct answer must be (ii).

(c) As we have 4 experimental runs, there must be 4 residuals. The residuals always sum to zero. Thus the correct answer must be (iv).