CHALMERS | UNIVERSITY OF GOTHENBURG

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Statistical Inference Principles – Spring 2016

Assignment 2

1. Let V be a Cauchy distributed random variable with pdf given by $f_V(x) = 1/\sqrt{2}\pi \cdot (1+x^2/2)^{-1}$ for $x \in \mathbb{R}$. Furthermore, let Y be a standard normal distributed random variable, i.e., the pdf of Y is given by $f_Y(x) = 1/\sqrt{2\pi}\exp(-x^2/2)$, $x \in \mathbb{R}$.

(a) Calculate $M := \sup_{y\in\mathbb{R}} \frac{f_Y(y)}{f_V(y)}$. Hint: Use the series expansion of the exponential function.

(b) Implement the Acceptance/Rejection Algorithm in your preferred programming language. Generate 100, 1.000, and 10.000 samples and plot the corresponding histograms. Compare them with the density of the standard normal distribution.

2. Let $(X_i, i = 1,\dots,n)$ be a random sample from a population with pdf or pmf $f(\cdot|\theta)$ from an exponential family given by

$$f(x|\theta) := h(x)c(\theta)\exp(\sum_{i=1}^{k} w_i(\theta)t_i(x)),$$

where $\theta := (\theta_1,\dots,\theta_d)$, $d \le k$. Show that

$$T(X) := (\sum_{j=1}^{n} t_1(X_j),\dots,\sum_{j=1}^{n} t_k(X_j))$$

is a sufficient statistic for θ.

3. Let (X_1, X_2) be a random sample of the discrete distribution given by

$$P_\theta(X_1 = \theta) = P_\theta(X_1 = \theta+1) = P_\theta(X_1 = \theta+2) = \frac{1}{3}.$$

Set $R := X_{(2)} - X_{(1)}$ and $M := (X_{(1)} + X_{(2)})/2$.

(a) Show that (R, M) is a minimal sufficient statistic for θ.

(b) Furthermore, show that R is an ancillary statistic, i.e., that the distribution of R does not depend on θ.

Deadline: Monday, February 1, 2016
Webpage: `http://www.math.chalmers.se/Stat/Grundutb/GU/MSF100/S16/`
Requirement: 50% of the exercises solved, two presentations in the exercise class