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Statistical Inference Principles – Spring 2016

Assignment 4

1. Suppose that W is an unbiased estimator of $\tau(\theta)$, and U is an unbiased estimator of 0. Show that if, for some θ_0, $\mathsf{Cov}_{\theta_0}(W,U) \neq 0$, then W cannot be the best unbiased estimator of $\tau(\theta)$.

2. Let $(X_i, i = 1,\dots,n)$ be a random sample of $\mathcal{N}(\theta,1)$-distributed random variables.

(a) Show that the best unbiased estimator of θ^2 is $\bar{X}^2 - 1/n$.

(b) Calculate the variance of $\bar{X}^2 - 1/n$ and show that it is greater than the Cramér–Rao Bound.

3. Let $(X_i, i = 1,\dots,n)$ be a random sample from a population with pdf

$$f(x|\theta) := \theta^x(1-\theta)^{1-x},$$

where $x \in \{0,1\}$ and $\theta \in [0,1/2]$. On Assignment 3, Task 2 you have computed the method of moments estimator and the MLE of θ.

a) Find the mean squared error of each of the estimators theoretically.

b) Compute the mean squared errors by sampling. Do they agree with the theory?

c) Additionally simulate mean squared errors for different sizes of data samples (e.g., a sequence of the form $(2^n, n = 1,\dots,N)$ might be convenient) and show the results for both estimators in a convergence plot. Try to fit a convergence order. A good way to see the quality of the fitted order is to use a loglog plot.

Deadline: Monday, February 15, 2016
Webpage: `http://www.math.chalmers.se/Stat/Grundutb/GU/MSF100/S16/`
Requirement: 50% of the exercises solved, two presentations in the exercise class