CHALMERS | UNIVERSITY OF GOTHENBURG

Annika Lang

Statistical Inference Principles – Spring 2016

Assignment 5

1. Let $(X_i, i = 1, \ldots, n)$ be a random sample of $\mathcal{N}(\theta, \sigma^2)$-distributed random variables. Let the prior of θ be $\mathcal{N}(\mu, \tau^2)$-distributed and σ^2, μ, and τ^2 be known. Implement with the results from Exercise 3 on Assignment 3 the Bayesian test discussed in Example 8.2.7 in the lecture. How are your acceptance rates influenced by the prior? Find a way to verify your observations graphically or with computed numbers.

2. **a)** Show that the family of normal distributions $\mathcal{N}(\theta, \sigma^2)$ with σ^2 known has a monotone likelihood ratio.

b) Suppose that the one-parameter exponential family $\{g(\cdot|\theta), \theta \in \Theta\}$ for the random variable T is given by $g(t|\theta) = h(t)c(\theta)\exp(w(\theta)t)$. Show that this family has a monotone likelihood ratio if w is an increasing function of θ. Give an example of such a family.

3. Prove the Neyman–Pearson Lemma (Theorem 8.3.12) for discrete random variables. Assume that level α is attainable.

Deadline: Monday, February 22, 2016
Webpage: `http://www.math.chalmers.se/Stat/Grundutb/GU/MSF100/S16/`
Requirement: 50% of the exercises solved, two presentations in the exercise class