CHALMERS | UNIVERSITY OF GOTHENBURG

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Statistical Inference Principles – Spring 2016

Assignment 6

1. **a)** Calculate a valid p-value for the following observation: For testing $H_0 : \theta \leq 1/2$ versus $H_1 : \theta > 1/2$, 7 successes are observed out of 10 Bernoulli trials.

b) Consider testing $H_0 : \theta \in \bigcup_{j=1}^k \Theta_j$ versus $H_1 : \theta \in \bigcap_{j=1}^k \Theta_j^c$, where k is finite. For each $j = 1, \dots, k$, let $p_j(X)$ denote a valid p-value for testing $H_{0,j} : \theta \in \Theta_j$ versus $H_{1,j} : \theta \in \Theta_j^c$. Let $p(x) := \max_{1 \leq j \leq k} p_j(x)$ for all $x \in \mathcal{X}$. Show first that $p(X)$ is a valid p-value for testing H_0 versus H_1. Furthermore, show that the level α test defined by $p(X)$ is the same as a level α intersection-union test defined in terms of individual tests based on the p-values $p_j(X)$, $j = 1, \dots, k$.

2. **a)** If T is a continuous random variable with cdf $F_T(\cdot|\theta)$ and $\alpha_1 + \alpha_2 = \alpha$, show that a level α acceptance region of the hypothesis $H_0 : \theta = \theta_0$ is $\{t \in \mathcal{T}, \alpha_1 \leq F_T(t|\theta_0) \leq 1-\alpha_2\}$, with associated $1-\alpha$ confidence set $\{\theta \in \Theta, \alpha_1 \leq F_T(t|\theta) \leq 1 - \alpha_2\}$.

b) Let $(X_i, i = 1, \dots, n)$ be a random sample with $\mathcal{N}(\theta, \sigma^2)$-distributed random variables, where σ^2 is known. For each of the following hypotheses, write out the acceptance region of a level α test and the $1 - \alpha$ confidence interval that results from inverting the test:

(i) $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$,
(ii) $H_0 : \theta \geq \theta_0$ versus $H_1 : \theta < \theta_0$,
(iii) $H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$.

c) Implement the interval estimator that corresponds to b).(i) for $\alpha = 0.05, 0.01, 0.005$ and test the amount of correct decisions for all three choices of α, where you are free to choose your favorite θ and σ^2.

3. Prove the following corollary from Theorem 9.3.2: Let f be a symmetric unimodal pdf. For a fixed value of $1 - \alpha$, of all intervals $[a, b]$ that satisfy $\int_a^b f(x)\,dx = 1 - \alpha$, the shortest is obtained by choosing a and b such that

$$\int_{-\infty}^{a} f(x)\,dx = \int_{b}^{\infty} f(x)\,dx = \frac{\alpha}{2}.$$

Deadline: Monday, February 29, 2016
Webpage: http://www.math.chalmers.se/Stat/Grundutb/GU/MSF100/S16/
Requirement: 50% of the exercises solved, two presentations in the exercise class