CHALMERS | UNIVERSITY OF GOTHENBURG

Annika Lang

Statistical Inference Principles – Spring 2018

Assignment 4

1. Suppose that W is an unbiased estimator of $\tau(\theta)$, and U is an unbiased estimator of 0. Show that if, for some $\theta_0 \in \Theta$, $\mathsf{Cov}(W, U|\theta_0) \neq 0$, then W cannot be the best unbiased estimator of $\tau(\theta)$.

2. Let X be a random sample of size n with $X_1 \sim \mathcal{N}(\theta, 1)$.

a) Show that the best unbiased estimator of θ^2 is $\bar{X}^2 - 1/n$.

b) Calculate the variance of $\bar{X}^2 - 1/n$ and show that it is greater than the Cramér–Rao Bound.

3. Let X be a random sample of size n with PDF determined from

$$f_{X_1}(x|\theta) := \theta^x(1-\theta)^{1-x},$$

where $x \in \{0, 1\}$ and $\theta \in [0, 1/2]$. On Assignment 3, Task 2 you have computed the method of moments estimator and the MLE of θ.

a) Find the mean squared error of each of the estimators theoretically.

b) Compute the mean squared errors by sampling. Do they agree with the theory?

c) Additionally simulate mean squared errors for different sizes of data samples (e.g., a sequence of the form $(2^n, n = 1, \ldots, N)$ might be convenient) and show the results for both estimators in a convergence plot. Try to fit a convergence order. A good way to see the quality of the fitted order is to use a loglog plot.

4. Let X be a random sample of size n from a Bernoulli(θ) distribution. One can show that the MLE of θ^2 is

$$S_n := \bar{X}^2.$$

Let us define a new estimator

$$T_n := n\, S_n - \frac{n-1}{n} \sum_{i=1}^{n} S_n^{(i)},$$

Please turn!

where

$$S_n^{(i)} := \left((n-1)^{-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} X_j \right)^2 .$$

a) Show that S_n is a biased estimator of θ^2.

b) Compute T_n explicitly.

c) Show that T_n is an unbiased estimator of θ^2.

d) Is T_n the best unbiased estimator? If yes, prove it, else find the best unbiased estimator of θ^2.

Deadline: Thursday, February 15, 2018, send an email before 14.30 with a list of solved problems.
Webpage: `http://www.math.chalmers.se/Stat/Grundutb/GU/MSF100/S18/`
Requirement: 75% of the exercises solved, two presentations in the exercise class