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Statistical Inference Principles – Spring 2018

Assignment 5

1. Assume that the family of PDFs $\{g_T(\cdot|\theta), \theta \in \Theta\}$ of T has a nondecreasing monotone likelihood ratio. Show that for any $c \in \mathbb{R}$ if $\theta_1 \leq \theta_2$ then

$$P(T > c|\theta_1) \leq P(T > c|\theta_2).$$

2. Let X be a random sample of size n with $X_1 \sim \mathcal{N}(\mu, \sigma^2)$ and assume that σ^2 is known. Consider

$$H_0 : \mu \leq \mu_0 \quad \text{versus} \quad H_1 : \mu > \mu_0$$

for some fixed $\mu_0 \in \mathbb{R}$. Derive the LRT statistic for this hypothesis by giving the rejection region.

3. Let us consider the hypothesis

$$H_0 : \theta \in \Theta_0 \quad \text{versus} \quad H_1 : \theta \in \Theta_0^c,$$

where $\Theta_0 := \bigcap_{\gamma \in \Gamma} \Theta_\gamma$ and Γ is some index set.

Let further λ_γ denote the LRT statistic for

$$H_{0,\gamma} : \theta \in \Theta_\gamma \quad \text{versus} \quad H_{1,\gamma} : \theta \in \Theta_\gamma^c.$$

Define $T(x) := \inf_{\gamma \in \Gamma} \lambda_\gamma(x)$ and form the union-intersection test with rejection region

$$R := \{x \in \mathcal{X}, \lambda_\gamma(x) < c \text{ for some } \gamma \in \Gamma\} = \{x \in \mathcal{X}, T(x) < c\}$$

for some fixed $c \in \mathbb{R}$. Also consider the usual LRT with rejection region $\{x \in \mathcal{X}, \lambda(x) < c\}$ of H_0 vs. H_1.

a) Show that $T(x) \geq \lambda(x)$ for all $x \in \mathcal{X}$.

b) Denote by β_T the power function based on $T(X)$ and by β_λ the power function obtained from the LRT based on λ. Show that $\beta_T(\theta) \leq \beta_\lambda(\theta)$ for all $\theta \in \Theta$.

c) Show that if the LRT for H_0 vs. H_1 is a level α test, then the test based on $T(X)$ is also a level α test.

Please turn!

4. **a)** Show that the family of normal distributions $\mathcal{N}(\mu, \sigma^2)$ with σ^2 known has a monotone likelihood ratio.

b) Suppose that the one-parameter exponential family $\{g(\cdot|\theta), \theta \in \Theta\}$ for the random variable T is given by $g(t|\theta) = h(t)c(\theta)\exp(w(\theta)t)$. Show that this family has a monotone likelihood ratio if w is an increasing function of θ. Give an example of such a family.

Deadline: Thursday, February 22, 2018, send an email before 14.30 with a list of solved problems.
Webpage: `http://www.math.chalmers.se/Stat/Grundutb/GU/MSF100/S18/`
Requirement: 75% of the exercises solved, two presentations in the exercise class