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Statistical Inference Principles – Spring 2018

Assignment 6

1. **a)** Calculate a valid p-value for the following observation: For testing $H_0 : \theta \leq 1/2$ versus $H_1 : \theta > 1/2$, 7 successes are observed out of 10 Bernoulli trials.

b) Consider testing $H_0 : \theta \in \bigcup_{j=1}^{k} \Theta_j$ versus $H_1 : \theta \in \bigcap_{j=1}^{k} \Theta_j^c$, where k is finite. For each $j = 1, \ldots, k$, let $p_j(X)$ denote a valid p-value for testing $H_{0,j} : \theta \in \Theta_j$ versus $H_{1,j} : \theta \in \Theta_j^c$. Let $p(x) := \max_{1 \leq j \leq k} p_j(x)$ for all $x \in \mathcal{X}$. Show first that $p(X)$ is a valid p-value for testing H_0 versus H_1. Furthermore, show that the level α test defined by $p(X)$ is the same as a level α intersection-union test defined in terms of individual tests based on the p-values $p_j(X)$, $j = 1, \ldots, k$.

2. If T is a continuous random variable with cdf $F_T(\cdot|\theta)$ and $\alpha_1 + \alpha_2 = \alpha$, show that a level α acceptance region of the hypothesis $H_0 : \theta = \theta_0$ is $\{t \in \mathcal{T}, \alpha_1 \leq F_T(t|\theta_0) \leq 1 - \alpha_2\}$, with associated $1 - \alpha$ confidence set $\{\theta \in \Theta, \alpha_1 \leq F_T(t|\theta) \leq 1 - \alpha_2\}$.

3. **a)** Let X be a random sample of size n with $X_1 \sim \mathcal{N}(\mu, \sigma^2)$, where σ^2 is assumed to be known. For each of the following hypotheses, write out the acceptance region of a level α test and the $1 - \alpha$ confidence interval that results from inverting the test:

(i) $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$,

(ii) $H_0 : \mu \geq \mu_0$ versus $H_1 : \mu < \mu_0$,

(iii) $H_0 : \mu \leq \mu_0$ versus $H_1 : \mu > \mu_0$.

b) Implement the interval estimator that corresponds to a).(i) for $\alpha = 0.05, 0.01, 0.005$ and test the amount of correct decisions for all three choices of α, where you are free to choose your favorite μ_0 and σ^2.

4. Let f be a symmetric unimodal PDF. Show that for a fixed value of $1 - \alpha$, of all intervals $[a, b]$ that satisfy $\int_a^b f(x)\,\mathrm{d}x = 1 - \alpha$, the shortest is obtained by choosing a and b such that

$$\int_{-\infty}^{a} f(x)\,\mathrm{d}x = \int_{b}^{\infty} f(x)\,\mathrm{d}x = \frac{\alpha}{2}.$$

Deadline: Thursday, March 1, 2018, send an email before 14.30 with a list of solved problems.
Webpage: `http://www.math.chalmers.se/Stat/Grundutb/GU/MSF100/S18/`
Requirement: 75% of the exercises solved, two presentations in the exercise class