Formula sheet for "Linear Statistical Models"

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[No need to print this sheet. It will be provided at the exam.]

Here follow some properties of expectation, variance, covariance and correlations of random variables. We used most of them during the course. Perhaps one or two relations were not used but are reported for completeness.

Table 1: Let Q, W and Z be random variables. a and b are constant (i.e. not random) scalar quantities. $\boldsymbol{A}$ and $\boldsymbol{B}$ are constant matrices. $E(\cdot)$ denotes expectation, $Var(\cdot)$ denotes variance and $Cov(\cdot)$ denotes covariance. $\rho(\cdot)$ denotes correlation. $'$ denotes transposition.

$E(a) = a$
$E(a \cdot W) = a \cdot E(W)$
$E(a \cdot W \pm b \cdot Z) = a \cdot E(W) \pm b \cdot E(Z)$
$Var(W) = E(W^2) - (E(W))^2 = E(W - E(W))^2$
$Var(a \cdot W \pm b \cdot Z) = a^2 \cdot Var(W) + b^2 Var(Z) \pm 2a \cdot b \cdot Cov(W, Z)$
$Var(a) = 0$
$Var(aW \pm b) = a^2 Var(W)$
$Var(\boldsymbol{A} \cdot W) = \boldsymbol{A} \cdot Var(W) \cdot \boldsymbol{A}'$
$Cov(W, Z) = E[(W - E(W))(Z - E(Z))] = E(WZ) - E(W)E(Z)$
$Cov(\boldsymbol{A} \cdot W, \boldsymbol{B} \cdot Z) = \boldsymbol{A} \cdot Cov(W, Z) \cdot \boldsymbol{B}'$
$Cov(W, Z) = 0$ if W and Z are independent.
$Cov(a + W, b + Z) = Cov(W, Z)$.
$Cov(a \cdot W, b \cdot Z) = ab \cdot Cov(W, Z)$.
$Cov(Q + W, Z) = Cov(Q, Z) + Cov(W, Z)$.
$Cov(W, W) = Var(W)$.
$\rho(W, Z) = \frac{Cov(W,Z)}{\sqrt{Var(W) \cdot Var(Z)}}$