Written examination in MVE170/MSG800
BASIC STOCHASTIC PROCESSES
December 14, 2009, 8:30 - 11:30 am
Lecturer and on duty: Rossitza Dodunekova, tel. 772 3534
Allowed material: The handbook *Beta*

Problem 1.

(a) Give two different definitions of the Poisson process.

(b) State without proof the memoryless property of the process.

(c) Formulate the PASTA property and comment on it. 3p

Problem 2.

(a) Define the homogeneous discrete-time Markov chain $\{X_n,\ n = 0,\ 1,\ \ldots\}$ and its n-step transition probabilities $p_{ij}^{(n)}$.

(b) State and prove the Chapman-Kolmogoroff equations for the transition probabilities of the process. 5p

Problem 3. An information centre has one attendant; people with questions arrive according to a Poisson process with rate λ. A person who finds n other customers present upon arrival joins the queue with probability $1/(n+1)$ for $n = 0,\ 1,\ \ldots$ and goes elsewhere otherwise. The service times of the persons are independent random variables having an exponential distribution with mean $1/\mu$.

(a) Formulate a continuous-time Markov chain to analyse the number of persons present at the information centre and specify the transition rate diagram.

(b) Find the equilibrium distribution of the process.

(c) What is the long-run fraction of persons with request who actually join the queue? Explain your answer.

(d) What is the long-run average number of persons served per time unit? Explain.

7p