Basic stochastic processes

Take home re-examination in April 2010

Day assigned: April 8, 10:00 am.
Due date: April 9, 10:00 am

- Solutions to the take home examination are to be submitted electronically.

- The take home examination is a strictly individual assignment. Submissions that bear signs of being collective efforts will be disregarded. Answers without explanations will be disregarded as well.

Problem 1. A Bernoulli trial results in a success with probability p and in a failure with probability $1-p$, where $0 < p < 1$. Suppose the Bernoulli trial is repeated indefinitely with each repetition independent of all others. Let X_n be a "success runs" Markov chain with a state space $I = \{0, 1, 2, \dots\}$, where $X_n = 0$ if the $n-th$ trial results in a failure and $X_n = j$ if $X_{n-j} = 0$ and trials $n-j+1, \dots, n$ have resulted in a success. Find the one-step transition matrix of the Markov chain. Show that all states are recurrent. 5p

Problem 2. In the beginning of each time unit a job arrives at a conveyor with a single work station. The workstation can process only one job at a time and has a buffer for waiting jobs, that can hold at most K jobs. Any arriving gob that finds the buffer full is lost. The processing times are independent and have exponential distribution with mean $1/\mu$. It is assumed that $\mu > 1$. Define a Markov chain to analyse the number of jobs in the buffer just prior to the arrival epochs of new jobs and specify the one-step transition probabilities. Show how to calculate the long-run fraction of lost jobs and the long-run fraction of time the workstation is busy. 5p

Problem 3. An information centre has one attendant; people with questions arrive according to a Poisson process with rate λ. A person who finds n other customers present upon arrival joins the queue with probability $1/(n+1)$ for $n = 0, 1, \dots$ and goes elsewhere otherwise. The service times of the persons are independent random variables having an exponential distribution with mean $1/\mu$. Verify that the equilibrium distribution of the number of persons present at the information centre is a Poisson distribution with mean λ/μ. What is the long-run fraction of persons with request who actually join the queue? What is the long-run average number of persons served per time unit? 5p