Solution to the written examination in MVE170 and MSG800

Basic stochastic processes

April 6, 2010, 8:30 - 11:30 am.

Problem 1. You wish to cross a one-way traffic road on which cars drive at a constant speed and pass according to a Poisson process with rate λ. You can only cross the road when no car has come round the corner for c time units. Suppose you arrive at a random moment. Find the probability distribution and the expectation of the number of passing cars before you can cross the road.

5p

Solution

Let X denote the number of passing cars before you cross the road. By the memoryless property of the Poisson process,

$$P(X = n) = (1 - e^{-\lambda c})^n e^{-\lambda c}, \quad n = 0,\, 1,\, 2 \ldots.$$

Hence X is a geometric rv with parameter $e^{-\lambda c}$. The average number of passing cars before you can cross the road is then $E[X] = e^{\lambda c}(1 - e^{-\lambda c})$.

Problem 2. [See the book]

(a) Give a definition of a renewal process $\{N(t),\, t \geq 0\}$ and its renewal function $M(t)$.

(b) For $n = 1,\, 2,\, \ldots$ let $F_n(t)$ be the distribution function of the renewal time S_n. Give a formula relating these functions and the function $M(t)$ and prove it.

(c) Let μ be the average interocurrence time of the process. Fix $t > 0$ and consider the excess variable $\gamma_t = S_{N(t)+1} - t$. Prove that

$$E[\gamma_t] = \mu[1 + M(t)] - t$$

5p

Problem 3. [See the book]

(a) Describe the M/M/1 queuing system. For any $t \geq 0$, let $X(t) =$ the number of customers present at time t. Derive the infinitesimal transition rates of the process and sketch the state diagram.

(b) Explain, under what assumption has the process equilibrium probabilities and compute these probabilities.

(c) Explain the formula that can be used to compute the long-rum average number of customers in queue and compute this number. What is the long-run fraction of customers who find j other customers present upon arrival?

5p