

MSG800/MVE170 Basic Stochastic Processes Fall 2012

Exercise Session 3, Thursday 15 November

Chapter 5 in Hsu's book (continued)

Solved problems. Problems 5.30, 5.32, 5.35, 5.49, 5.55 and 5.60 in Hsu's book.

Supplementary problems for own work. Problems 5.92, 5.93, 5.95, 5.98, 5.100 and 5.101 in Hsu's book.

Computer problem for own work. Let $\{W(t)\}_{t \geq 0}$ be a Wiener process with $\sigma^2 = \text{Var}\{W(1)\} = 1$. For a real constant $\varepsilon > 0$, consider the differential ratio process $\Delta_\varepsilon = \{\Delta_\varepsilon(t)\}_{t > 0}$ given by

$$\Delta_\varepsilon(t) = \frac{W(t+\varepsilon) - W(t)}{\varepsilon} \quad \text{for } t > 0.$$

For $s > 0$ and $t \in \mathbb{R}$ (the latter of which has an absolute value small enough to make $s+t \geq 0$), show that the autocorrelation function

$$R_{\Delta_\varepsilon}(t) = R_{\Delta_\varepsilon}(s, s+t) = \mathbf{E}\{\Delta_\varepsilon(s)\Delta_\varepsilon(s+t)\}$$

of Δ_ε is a triangle like function that depends on the difference t between $s > 0$ and $s+t \geq 0$ only. Further, show that $R_{\Delta_\varepsilon}(t) \rightarrow \delta(t)$ (Dirac's δ -function) as $\varepsilon \downarrow 0$. Simulate a sample path of $\{\Delta_\varepsilon(t)\}_{t \in (0,1]}$ for a really small $\varepsilon > 0$ (recall that W has independent increments) and plot a graph of that sample path. Discuss the claim that the (non-existing in the usual sense) derivative process $\{W'(t)\}_{t \geq 0}$ of W is white noise.