## MSG800/MVE170 Basic Stochastic Processes Fall 2013 List of Errata for Hsu's book, Version 2 November 2013

**Equation 3.14.** It should be  $\lim_{x\to\infty}$  instead of  $\lim_{y\to\infty}$ .

**Figure 3.4.** It should be P(Y=0|X=1) instead of P(Y=1|X=1) on the diagonal.

**Problem 4.100 in the Second Edition** (Problem 4.80 in the first). X'' should be  $X^n$ .

**Equation 5.28** is wrong as is the proof of (another version of) that formula in Problem 5.25: It is the second equality in the three line equation in the middle of the problem text that is erroneous. To see this we send  $x_{n-1} \to \infty$  on both sides of the equation to obtain

$$F_X(x_1,\ldots,x_{n-2},x_n;t_1,\ldots,t_{n-2},t_n) = F_X(x_n;t_n) \times F_X(x_1,\ldots,x_{n-2};t_1,\ldots,t_{n-2}),$$

which is to say that  $X(t_n)$  is independent of  $X(t_1), ..., X(t_{n-2})$ . This is absurd as this need not hold at all for a Markov process, as is e.g., exemplified by a Poisson process.

Equations 5.21-5.22. The correct definition of wide-sense stationarity is that the two conditions (5.21) and (5.22) hold.

On the line before Equation 5.51. (Prob. 5.74) should be (Prob. 5.94).

**Lemma 5.8.1 in the Second Edition.**  $T_1$  and  $T_2$  instead of  $n_1$  and  $n_2$ , respectively.

Equation 5.121 in the Second Edition.  $\cos \omega \tau$  instead of  $\cos \omega t$  on the middle row.

**Problem 5.37.** It should be  $gcd\{2,4,6,\ldots\}$  instead of  $gcd\{2,5,6,\ldots\}$ .

**Problem 5.49.** Four occurances of  $X(t+\Delta t)-X(0)$  should be  $X(t+\Delta t)-X(t)$ .

Equation 5.202 in the Second Edition. It should be  $\sigma$ 's instead of a's in  $K_X$ .

**Problem 5.70 in the Second Edition.** The alternative possible value of  $X_i$  should not be 0 but -1 with probability q = 1 - p. Such a random variable is not Bernoulli distributed.

**Problem 5.72 in the Second Edition.** On row 11  $\frac{n(n-1)}{2}$  should be n(n-1) and  $\frac{n(n+3)}{2}$  should be n(n+1).

**Problem 5.74 in the Second Edition.** On row 7 it should be  $\frac{n+2-k}{n+2}$  instead of  $\frac{n+2+k}{n+2}$ .

Problem 5.77 in the Second Edition. On row 4 of the problem it should be

$$\mathbf{E}\{|X_n|\} = \mathbf{E}\{|\mathbf{E}\{X|F_n\}|\} \le \mathbf{E}\{\mathbf{E}\{|X||F_n\}\} = \mathbf{E}\{|X|\} < \infty.$$

The initial condition should be  $\mathbf{E}\{|X|\} < \infty$  instead of  $\mathbf{E}\{X\} < \infty$  accordingly.

**Problem 5.78 in the Second Edition.** Theorem 5.82 should be Theorem 5.8.2.

Page 266 in the Second Edition. On row 11  $s_n$  should be  $S_n$  and on row -4 the first  $\frac{a}{a+b}$  should be  $\frac{b}{a+b}$ .

**Problem 5.82 in the Second Edition.** It should be  $X(t) - \lambda t$  instead of  $x(t) - \lambda t$ .

**Problem 6.8 b in the Second Edition.** On the first row of the solution it should be  $\partial^2 R_X(s-t)/\partial t\partial s = -d^2 R_X(\tau)/d\tau^2$ .

Page 288 in the Second Edition. On row 8 it should be  $E[X(s)X(\beta)]$  and on row 12 the second  $s^3$  should be  $s^2$ .

Equation 6.137 in the Second Edition. It should be  $R_{X'}(t,s)$  instead of  $R_X(t,s)$ .

Page 295 in the Second Edition. On the last line  $\frac{a}{(a^2+b^2)b}$  should be  $\frac{a}{(a^2-b^2)b}$ .

**Equation 9.20.** In the sum it should be  $(s\rho)^n$  instead of just  $(s\rho)$ .

Equations 9.22 in the Second Edition. Correct sp to sp.

Equations 9.36. Correct  $\rho_0$  to  $p_0$ .

**Equation 9.39.** On the right-hand side  $L_q$  should be divided by  $\lambda_e = \lambda (1 - p_K)$ .

**Problem 9.13.** In the solution to Task a (9.16) should be (9.17). In the final evaluation of  $W_q$  Hsu has forgotten to subtract  $-1/\mu = -3$ , but the result 6.39 min is still right.

**Problem 9.16.** The "balance equation" (9.2)  $L = \lambda W$  holds for all steady-state queues provided that we set the total time spent in the system to zero for customers arriving when the system is full for queues with  $K < \infty$ , i.e., if we interpret the fact that customers arriving to a full system bounces away as that they spend zero time in system.

If we instead (as is custom and as is done in the book) define W as the expected total time spent in the system for customers that really join the system, then we must divide the W from the previous paragraph by  $1-p_K$  to get this W. And then we have Equation 9.31  $L = \lambda (1-p_K) W$  for this alternative (and customary) definition of W.

In the solution to Problem 9.16 the sum in the second equation from the end should only run to K-1, giving  $W_q = (L-K\,p_K)/\mu$ . The sum in the last equation should also only run to K-1 giving  $W = (L+1-(K+1)\,p_K)/\mu$ . Now, here we are dealing with the first definition of W from above, i.e., we assign zero time in the system for bouncing customers and let that zero influence the mean value W, so that  $L = \lambda W$  for this W.

As we want the customary definition of W to be the expected total time spent in the system for customers that really joins the system we have to divide the W obtained in the previous paragraph by  $1-p_K$  giving the following correct version of Equation 9.59:  $W = (L+1-(K+1)\,p_K)/(\mu\,(1-p_K)).$  The corresponding correct version of Equation 9.58 is  $W_q = (L+1-(K+1)\,p_K)/(\mu\,(1-p_K)) - 1/\mu = (L-K\,p_K)/(\mu\,(1-p_K)).$ 

**Problem 9.18 in the Second Edition.** On row 3 from the bottom  $p^m$  should be  $\rho^m$ .

**Problem 9.29.** In Task a Hsu has used the faulty Equation 9.59 (see above) to obtain  $W=(L+1)/\mu=\ldots=0.336$  hours = 20.15 minutes, while a correct answer comes from using the correct version of that formula from above  $W=(L+1-(K+1)\,p_K)/(\mu\,(1-p_K))=(L-K\,p_K)/(\mu\,(1-p_K))+1/\mu=7/57+1/6=0.289$  hours = 17.4 minutes (or from using Equation 9.31). The answer to Task b is  $W_q=W-1/\mu=W-10$  minutes = 7.4 minutes.

**Problem 9.30.** The correct answer is 1.

**Table B-2.** In Formula 11 the right hand side should be  $X(\omega) = (\pi/a) e^{-a|w|}$ .