MSG800/MVE170 Basic Stochastic Processes Fall 2013

Exercise Session 1

Equations (equation numbers) and problems (problem numbers) in Chapter 5 of the Second Edition of Hsu's book do not agree completely with those in the First Edition: The difference is that the Second Edition contains a new Section 5.8 about martingales as compared with the First Edition that has been inserted before the problems (exercises). As a result of that additional corresponding problems also have been added to the Second Edition.

In numbers: Equations 5.1-5.65 and Problems 5.1-5.62 agree in both editions of the book (except that equation numbers in the problems in the Second Edition are 23 units higher that in the First Edition due to the new Section 5.8). Equations 5.66-5.88 in Section 5.8 and Equations 5.208-5.226 in the corresponding set of problems of the Second Edition are not present in the First Edition, while Equations 5.89-5.207 in the Second Edition correspond to Equations 5.66-5.184 in the First Edition. Problems 5.63-5.82 and Supplementary Problems 5.103-5.106 in the Second Edition are not present in the First Edition, while Supplementary Problems 5.83-5.102 in the Second Edition correspond to Supplementary Problems 5.63-5.82 in the First Edition.

Sections 5.1-5.4 Hsu's book

Solved problems. Problems 5.10, 5.13, 5.21, 5.22, 5.23 and 5.26 in Hsu's book.

Problems for own work. Problems 5.83, 5.84, 5.85 and 5.86 in the Second Edition of Hsu's book (Problems 5.63, 5.64, 5.65 and 5.66 in the First Edition).

Computer problem. Let $\{W(t)\}_{t\geq 0}$ be a so called Wiener process, that is, a process with stationary and independent normal distributed increments (we will learn more about this process in Section 5.7 in Hsu's book). Recall from Problem 5.23 in Hsu's book that $\mathbf{Cov}\{W(s), W(t)\} = \sigma^2 \min\{s,t\}$ where $\sigma^2 = \mathbf{Var}\{W(1)\}$.

For a real constant $\varepsilon > 0$, consider the differential ratio process $\Delta_\varepsilon = \{\Delta_\varepsilon(t)\}_{t>0}$ given by

$$\Delta_\varepsilon(t) = \frac{W(t+\varepsilon) - W(t)}{\varepsilon} \quad \text{for } t > 0.$$

For $s > 0$ and $t \in \mathbb{R}$ (the latter of which has an absoulte value small enough to make

$s+t \geq 0$), show that the autocorrelation function

$$R_{\Delta_\varepsilon}(t) = R_{\Delta_\varepsilon}(s, s+t) = \mathbf{E}\{\Delta_\varepsilon(s)\Delta_\varepsilon(s+t)\}$$

of Δ_ε is a triangle like function that depends on the difference t between $s > 0$ and $s+t \geq 0$ only. Further, show that $R_{\Delta_\varepsilon}(t) \to \delta(t)$ (Dirac's δ-function) as $\varepsilon \downarrow 0$. Simulate a sample path of $\{\Delta_\varepsilon(t)\}_{t\in(0,1]}$ for a really small $\varepsilon > 0$ (recall that W has independent increnments) and plot a graph of that sample path. Discuss the claim that the (non-existing in the usual sense) derivative process $\{W'(t)\}_{t\geq 0}$ of W is white noise.