

Calculation of the limit in Exercise 4 of Section 6.11 in G & S

```
In[2]:= Clear[G, mu, lambda, s, t];  
G[mu_, lambda_, s_, t_] := (mu*(1-s) - (mu - lambda*s) * Exp[-t*(lambda - mu)]) /  
  (lambda*(1-s) - (mu - lambda*s) * Exp[-t*(lambda - mu)]);
```

```
In[5]:= Limit[(G[mu, lambda, s, t] - G[mu, lambda, 0, t]) / (1 - G[mu, lambda, 0, t]),  
  t -> Infinity, Assumptions -> mu > lambda]
```

```
Out[5]= 
$$\frac{(\lambda - \mu) s}{-\mu + \lambda s}$$

```

Note that this is equal to

```
In[9]:= (1 - rho) * s / (1 - rho * s)
```

```
Out[9]= 
$$\frac{(1 - \text{rho}) s}{1 - \text{rho} s}$$

```

```
In[11]:= Simplify[Sum[s^n*(1-p)^(n-1)*p, {n, 1, Infinity}]]
```

```
Out[11]= 
$$\frac{p s}{1 + (-1 + p) s}$$

```

Hence we have a geometric distribution with parameter $1 - \text{rho}$