

# MSG800/MVE170 Basic Stochastic Processes Fall 2014

## Exercise Session 3

### Chapter 5 in Hsu's book (continued)

**Solved problems.** Problems 5.30, 5.32, 5.35, 5.49, 5.55 and 5.60 in Hsu's book.

**Supplementary problems** for own work. Problems 5.92, 5.93, 5.95, 5.98, 5.100 and 5.101 in Hsu's book.

**Computer problem** for own work. Let  $\{W(t)\}_{t \geq 0}$  be a Wiener process with  $\sigma^2 = \text{Var}\{W(1)\} = 1$ . For a real constant  $\varepsilon > 0$ , consider the differential ratio process  $\Delta_\varepsilon = \{\Delta_\varepsilon(t)\}_{t > 0}$  given by

$$\Delta_\varepsilon(t) = \frac{W(t+\varepsilon) - W(t)}{\varepsilon} \quad \text{for } t > 0.$$

For  $s > 0$  and  $t \in \mathbb{R}$  (the latter of which has an absolute value small enough to make  $s+t \geq 0$ ), show that the autocorrelation function

$$R_{\Delta_\varepsilon}(t) = R_{\Delta_\varepsilon}(s, s+t) = \mathbf{E}\{\Delta_\varepsilon(s)\Delta_\varepsilon(s+t)\}$$

of  $\Delta_\varepsilon$  is a triangle like function that depends on the difference  $t$  between  $s > 0$  and  $s+t \geq 0$  only. Further, show that  $R_{\Delta_\varepsilon}(t) \rightarrow \delta(t)$  (Dirac's  $\delta$ -function) as  $\varepsilon \downarrow 0$ . Simulate a sample path of  $\{\Delta_\varepsilon(t)\}_{t \in (0,1]}$  for a really small  $\varepsilon > 0$  (recall that  $W$  has independent increments) and plot a graph of that sample path. Discuss the claim that the (non-existing in the usual sense) derivative process  $\{W'(t)\}_{t \geq 0}$  of  $W$  is white noise.