

MSG800/MVE170 Basic Stochastic Processes Fall 2014

Exercise Session 7

Archetypical type-problems of typical type-exam-type for own work

Archetypical type-problem of typical type-exam-type 1. Consider a Markov chain $\{X_n\}_{n=0}^\infty$ with state space E and transition matrix P given by

$$E = \{0, 1, 2\} \quad \text{and} \quad P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix},$$

respectively, and let $E_i = \mathbf{E}\{\min\{n \geq 1 : X_n = i\} \mid X_0 = i\}$ for $i = 0, 1, 2$. Show that the chain has stationary distribution $[1/E_0 \ 1/E_1 \ 1/E_2]$.

Archetypical type-problem of typical type-exam-type 2. Let $\{W(t)\}_{t \geq 0}$ be a Wiener process and $\lambda > 0$ a constant. Show that $\{W(\lambda t)\}_{t \geq 0}$ is also a Wiener process.

Archetypical type-problem of typical type-exam-type 3. Let $\{N(t)\}_{t \geq 0}$ be a Poisson process with intensity $\lambda > 0$. Show that $\{e^{-\lambda t} 2^{N(t)}\}_{t \geq 0}$ is a martingale wrt. the filtration F_s , $s \geq 0$, containing all information about the process values $\{N(r)\}_{r \leq s}$.

Archetypical type-problem of typical type-exam-type 4. Let $\{e_n\}_{n \in \mathbb{Z}}$ be uncorrelated zero-mean and unit variance random variables (i.e., discrete time white noise). Find the autocorrelation function of the process $\{X_n\}_{n \in \mathbb{Z}}$ given by $X_n = e_n + e_{n-1}/2$.

Archetypical type-problem of typical type-exam-type 5. Recall that differentiating a random process $\{X(t)\}_{t \in \mathbb{R}}$ corresponds to processing the process through a linear system with frequency response $H(\omega) = j\omega$. Show that the derivative of a WSS process X with autocorrelation function $R_{XX}(\tau) = e^{-|\tau|}$ has autocorrelation function $R_{X'X'}(\tau) = 2\delta(\tau) - e^{-|\tau|}$.

Archetypical type-problem of typical type-exam-type 6. Let $N(t)$ for $t \geq 0$ denote the total number of customers in a M/M/2/4 queuing system with exp(1)-distributed times between arriving customers as well as exp(1)-distributed service times. Assume that $N(0) = 0$ and let $\{T_n\}_{n=1}^\infty$ be the strictly increasing sequence of random times at which $\{N(t)\}_{t \geq 0}$ changes its values, that is, $T_{n+1} = \min\{t > T_n : N(t) \neq N(T_n)\}$ for $n \in \mathbb{N}$, with the convention $T_0 = 0$. Find the transition matrix P for the Markov chain $\{X_n\}_{n=0}^\infty$ with state space $E = \{0, 1, 2, 3, 4\}$ given by $X_n = N(T_n)$.