MSG800/MVE170 Basic Stochastic Processes

Written exam Friday 28 August 2015 $2-6$ pm

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Aids: Either two A4-sheets (4 pages) of hand-written notes (xerox-copies and/or computer print-outs are not allowed) or Beta (but not both these aids).

Grades: 12 points for grades 3 and G, 18 points for grade 4, 21 points for grade VG and 24 points for grade 5, respectively.

Motivations: All answers/solutions must be motivated. Good Luck!

Task 1. State a problem that argubly is very hard to solve analytically, but that can be solved in a rather straightforward manner by means of computer simulations. Supply a computer program (e.g., in terms of a flow chart or so called "pesudo-code") that solves the problem. **(5 points)**

Task 2. Find a (discrete time or continuous time) random process that is a Markov chain but not a martingale. **(5 points)**

Task 3. Find a (discrete time or continuous time) random process that is wide-sense stationary (WSS) but not strict-sense stationary. Also, is it possible for a strict-sense stationary random process not to be wide-sense stationary? **(5 points)**

Task 4. An LTI system is a relationship $y(t) = \{\mathbf{T}x\}(t)$ between an input signal $x(t)$ and the corresponding output signal $y(t)$ that is linear and time-invariant. For such a system in discrete time it holds that $\{\mathbf{T}x\}(t) = \sum_{i=-\infty}^{\infty} x(i)\, h(t-i)$, where $h(t) = \{\mathbf{T}\delta\}(t)$ is the impulse response. Prove this formula! **(5 points)**

Task 5. In a queueing system that basically is M/M/2/2 with $\lambda = \mu = 1$ one of the servers is ill and therefore performs only half a quick as usual, that is, has $\mu = 1/2$. When the system has one customer the healthy server is always empoyed. (In particular, when the system has two customers one of which is just having the service finished, then the healthy server continues with the service of the remaining customer, regardless of which server that started that service.) Find p_0, p_1 and p_2. **(5 points)**

Task 6. Find the distribution for the (random) time it takes an M/M/1/2 queueing system with $\lambda = \mu = 1$ to change its state from being full to being empty. **(5 points)**

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Solutions to written exam Friday 28 August

Task 1. See, e.g., the computational tasks of the course.

Task 2. In discrete time we may take a non-symmetric random walk and in continuous time a Poisson process (where in fact the latter is just a continuous time version of the former).

Task 3. A sequence of uncorrelated random variables with common expected values and common variances constitute a WSS discrete time process, but is not strict-sense stationary if the random variables are not identically distributed. A sequnce of independent identically distributed random variables with infinite variances constitute a strict-sense stationary discrete time process that is not WSS.

Task 4. We have $\{\mathbf{T}x\}(t) = \mathbf{T}\sum_{i=-\infty}^{\infty} x(i)\,\delta(t-i) = \sum_{i=-\infty}^{\infty} x(i)\,\{\mathbf{T}\delta\}(t-i) = \sum_{i=-\infty}^{\infty} x(i)\,h(t-i)$.

Task 5. By page 351 in Hsu's book we have $p_1 = (a_0/d_1)\,p_0$ and $p_2 = (a_0 a_1/(d_1 d_2))\,p_0$, where $a_0 = a_1 = \lambda = 1$, $d_1 = \mu = 1$ and $d_2 = 1 + 1/2 = 3/2$, giving $(p_0, p_1, p_2) = (p_0, p_0, (2/3)p_0) = (3/8, 3/8, 2/8)$.

Task 6. Let $X_1, X_2, X_3, \ldots$ be independent identically distributed random variables each of which is distributed like the sum of two independent random variables that are exponentially distributed with expected values 1 and 1/2, respectively. Further, let N be a discrete random variable independent of the X_n's such that $P(N = n) = 2^{-n}$ for $n = 1, 2, 3, \ldots$. Then the asked for random time is distributed like $\sum_{n=1}^{N} X_n$.